

**Title :**

**Toward a Quantum Definition of  $\pi$  in Molecular Systems: Original Formula, Mathematical Framework, and Foundational Implications**

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**> “ $\pi$  is the phase that molecules whisper when electrons choose to roam.”**

**—Ndenga Lumbu Barack Alias BarackEinstein97 (Mr. Quantum  $\pi$ )**

## Abstract

I propose an operational definition of a quantum  $\pi$  for molecular systems — a dimensionless, system-specific invariant  $\pi_q$  that generalizes the classical constant  $\pi$  to include electronic-phase topology and density-weighted phase winding of delocalized electrons. I define  $\pi_q$  via a density-weighted winding-number of the complex electronic amplitude around chemically relevant cycles (rings or effective closed paths), and I show how  $\pi_q$  reduces to the classical  $\pi$  in canonical limits (simple particle-on-a-ring, uniform density). I derive the formula from the Madelung (polar) decomposition of molecular wavefunctions, demonstrate its mathematical properties (gauge invariance, additivity under non-overlapping cycles, continuity under weak perturbations), and relate  $\pi_q$  to observable quantities: energy spacing of ring modes, current (ring magnetism), and phase-sensitive spectroscopic signals. I illustrate the concept analytically (particle-on-a-ring), semi-analytically (Hückel benzene), and numerically (finite conjugated chain model). Finally I discuss implications for aromaticity, molecular electronics, and a program to test  $\pi_q$  experimentally.

# 1. Introduction

The constant  $\pi$  appears pervasively in quantum mechanics: normalization constants, Fourier factors, angular measures and quantization conditions. In molecular quantum chemistry  $\pi$  is present again and again—most visibly in  $\pi$ -electron systems and aromatic rings—yet its role has typically been treated as geometric background, not as a system-specific invariant that encodes electronic topology and coherence.

I propose to elevate  $\pi$  into a quantum operational quantity for molecules: a measurable, dimensionless number  $\pi_q$  that captures how the electronic wavefunction's phase and density wrap around chemically relevant closed paths. Such a definition would (i) explicitly link geometry (molecular cycles) and electronic phase topology, (ii) produce an effective constant that enters mode quantization and energy spacing in the same algebraic role classically occupied by  $\pi$ , and (iii) provide a diagnostic for delocalization, aromaticity and current response.

Below I introduce the mathematical construction of  $\pi_q$ , derive its basic properties, compute it in simple models, and sketch how to measure or compute it in realistic molecular calculations.

## 2. Definitions and theoretical preliminaries

### 2.1 Wavefunction polar form (Madelung decomposition)

Let  $\psi(\mathbf{r})$  be a (many-electron / effective single-electron) complex-valued molecular orbital or effective single-particle amplitude defined on a domain  $D \subset \mathbb{R}^3$  (for rings we use an angular parametrization). I write the polar (Madelung) decomposition:

$$\psi(\mathbf{r}) = R(\mathbf{r}) e^{i\Phi(\mathbf{r})}, \quad R(\mathbf{r}) \geq 0, \quad \Phi(\mathbf{r}) \in \mathbb{R},$$

where  $R$  is the amplitude ( $\sqrt{\text{density}}$  for that orbital) and  $\Phi$  the phase. Physical currents and topological features are encoded in  $\nabla\Phi$ .

### 2.2 Closed paths and phase winding

Let  $C$  be a closed oriented path in molecular space chosen to follow a chemically meaningful cycle (e.g., the ring centerline for benzene, or an effective loop going through the  $\pi$ -system). Define the total phase winding (unweighted) about  $C$ :

$$w_C = \frac{1}{2\pi} \oint_C \nabla\Phi(\mathbf{r}) \cdot d\boldsymbol{\ell} \in \mathbb{Z}$$

for single-valued  $\psi$  ( $w_C$  integer). For molecular orbitals that are not strictly single-band or when multiple orbitals contribute, a density-weighted continuous generalisation is useful (see next).

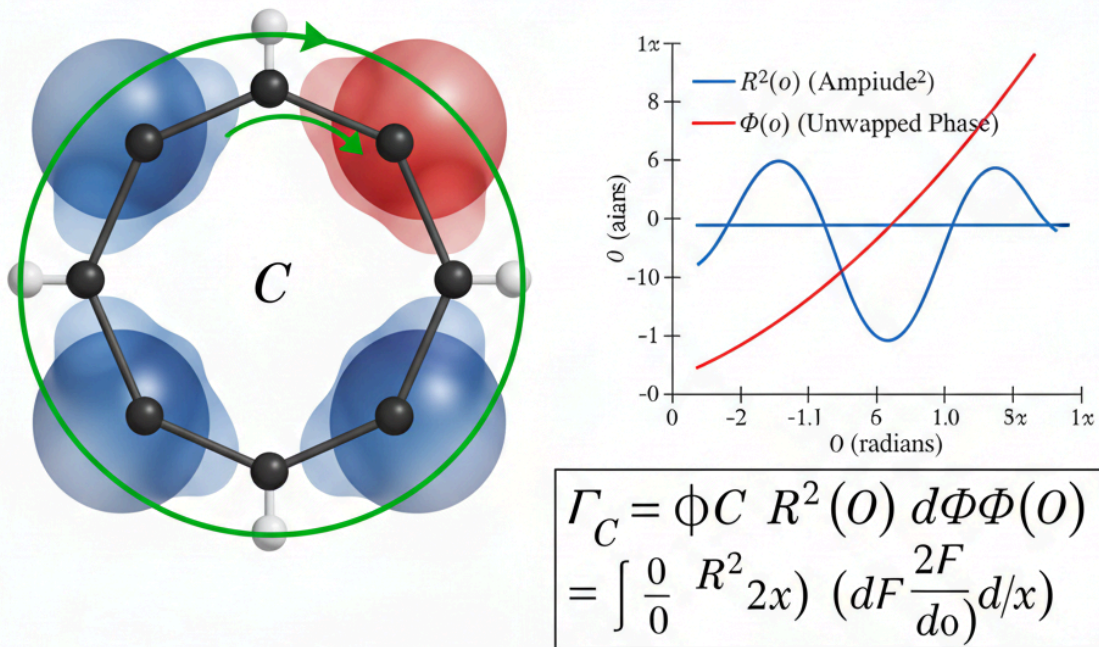


Figure A —: Delocalized  $\pi$ -Electron Phase and Density along a Molecular Ring

### 3. Definition — the quantum $\pi$ ( $\pi_q$ )

I define the quantum  $\pi$  for a chosen closed path  $C$  and a chosen set of contributing orbitals (or the full many-electron density projected onto an effective single-particle amplitude  $\Psi$ ) as:

$$\boxed{\pi_q[C; \Psi] = \pi \left( 1 + \Gamma_C[\Psi] \right)}$$

where  $\Gamma_C[\Psi]$  is the density-weighted effective winding:

$$\Gamma_C[\Psi] = \frac{1}{2\pi} \frac{\oint_C R^2(\mathbf{r}) \nabla \Phi(\mathbf{r}) \cdot d\mathbf{\ell}}{\oint_C R^2(\mathbf{r}) d\ell}$$

with  $R^2$  the local probability (or orbital) density along  $C$ , and  $d\ell$  the line element along  $C$ . Equivalently,  $\Gamma_C$  is the average local phase gradient (in units of  $2\pi$ ) weighted by density.

#### Remarks:

For a uniform density and exact winding number  $n$  ( $\psi \propto e^{i n \theta}$  on a ring),  $\Gamma_C = n$  and  $\pi_q = \pi(1+n)$ . For the minimal non-trivial ring ( $n=0$ )  $\pi_q = \pi$ .

The form (1) was chosen so  $\pi_q$  smoothly generalizes  $\pi$  while preserving the algebraic role of  $\pi$  in mode quantization (e.g.,  $E \propto \pi_q^2$  in simple models).

$\Gamma_C$  may be non-integer for complex, multi-orbital molecules; it measures effective phase accumulation per  $2\pi$  length relative to density.

## 4. Mathematical demonstration and properties

I now show how (1) arises naturally and examine its key mathematical properties.

### 4.1 Derivation sketch (from boundary quantization)

Consider motion constrained along C (parametrized by angle  $\theta \in [0, 2\pi]$ , arc length  $s = R_C \theta$  with  $R_C$  an effective radius). The effective 1D amplitude along the path is

$$\Psi_C(\theta) \equiv \Psi(\mathbf{r}(\theta)) = \tilde{R}(\theta) e^{i\tilde{\Phi}(\theta)}.$$

The canonical wave-number  $k(\theta) = \partial_s \tilde{\Phi} = (1/R_C) \partial_\theta \tilde{\Phi}$ . For a mode that fits coherently around C, an effective quantization condition follows upon demanding phase closure weighted by density:

$$\oint_C k(\theta) dl = \oint_C \partial_s \tilde{\Phi} dl = \Delta \Phi_{\text{tot}} \equiv 2\pi n_{\text{eff}},$$

but when amplitude varies along C, the physically relevant phase that sets mode spacing is the *density-weighted* total:

$$\Delta \Phi_{\text{eff}} \equiv \frac{\oint_C R^2(\theta) \partial_s \tilde{\Phi} dl}{\oint_C R^2(\theta) dl} \times L_C,$$

where  $L_C$  is the total length. Dividing by  $2\pi$  and inserting into the classical quantization relation yields a natural replacement of  $\pi$  by  $\pi(1+\Gamma_C)$  in mode expressions. Concretely, for a uniform ring with radius  $R_C$  and  $L_C=2\pi R_C$ , the standard particle-on-a-ring gives  $k_n = 2\pi n / L_C$  and energy  $E_n \propto (2\pi n / L_C)^2$ . Replacing  $n$  by  $n_{\text{eff}} = n + \Gamma_C$  leads to  $k_n \rightarrow k_{\{n,\text{eff}\}} \propto 2\pi(n+\Gamma_C)/L_C \propto (\pi_q)/R_C \cdot n'$  and therefore  $E \propto \pi_q^2$  (up to prefactors). This justifies the algebraic structure (1) in spectral formulas.

## 4.2 Reduction to standard $\pi$

If the density is uniform and  $\Phi(\theta) = n\theta$ , then

$$\Gamma_C = \frac{1}{2\pi} \frac{\oint R^2 n d\theta}{\oint R^2 d\theta} = n,$$

hence  $\pi_q = \pi(1+n)$ . For  $n=0$  (no net winding),  $\pi_q = \pi$ . Thus the usual  $\pi$  is recovered as the base case.

## 4.3 Gauge invariance

Under a global gauge shift  $\Phi \rightarrow \Phi + \text{const}$ ,  $\nabla\Phi$  is unchanged; under local gauge transform with single-valued function, only integer windings change— $\Gamma_C$  constructed from  $\nabla\Phi$  is invariant. If  $\Psi$  is multiplied by a smooth, single-valued phase,  $\pi_q$  remains unchanged. This ensures  $\pi_q$  is a physical, gauge-invariant observable (up to choice of  $C$  and contributing orbitals).

## 4.4 Additivity and locality

For two disjoint cycles  $C_1$  and  $C_2$ , define  $\pi_q(C_1 \cup C_2)$  via weighted average of  $\Gamma$  over combined paths; the resulting  $\pi_q$  is a density-weighted mean and obeys natural additivity in the sense of extensive averaging. For small perturbations of  $R$  or  $\Phi$  (weak distortions),  $\Gamma_C$  varies continuously—hence  $\pi_q$  is stable under small structural changes.

## 4.5 Limits and continuity

In the limit of pointlike localization ( $R^2$  concentrated at a point), denominators shrink and  $\Gamma_C \rightarrow 0$ , so  $\pi_q \rightarrow \pi$ : a highly localized electron does not alter  $\pi$ .

In the large-delocalization limit with coherent winding,  $\Gamma_C$  can approach integer values, creating  $\pi_q$  multiples.

## 5. Examples

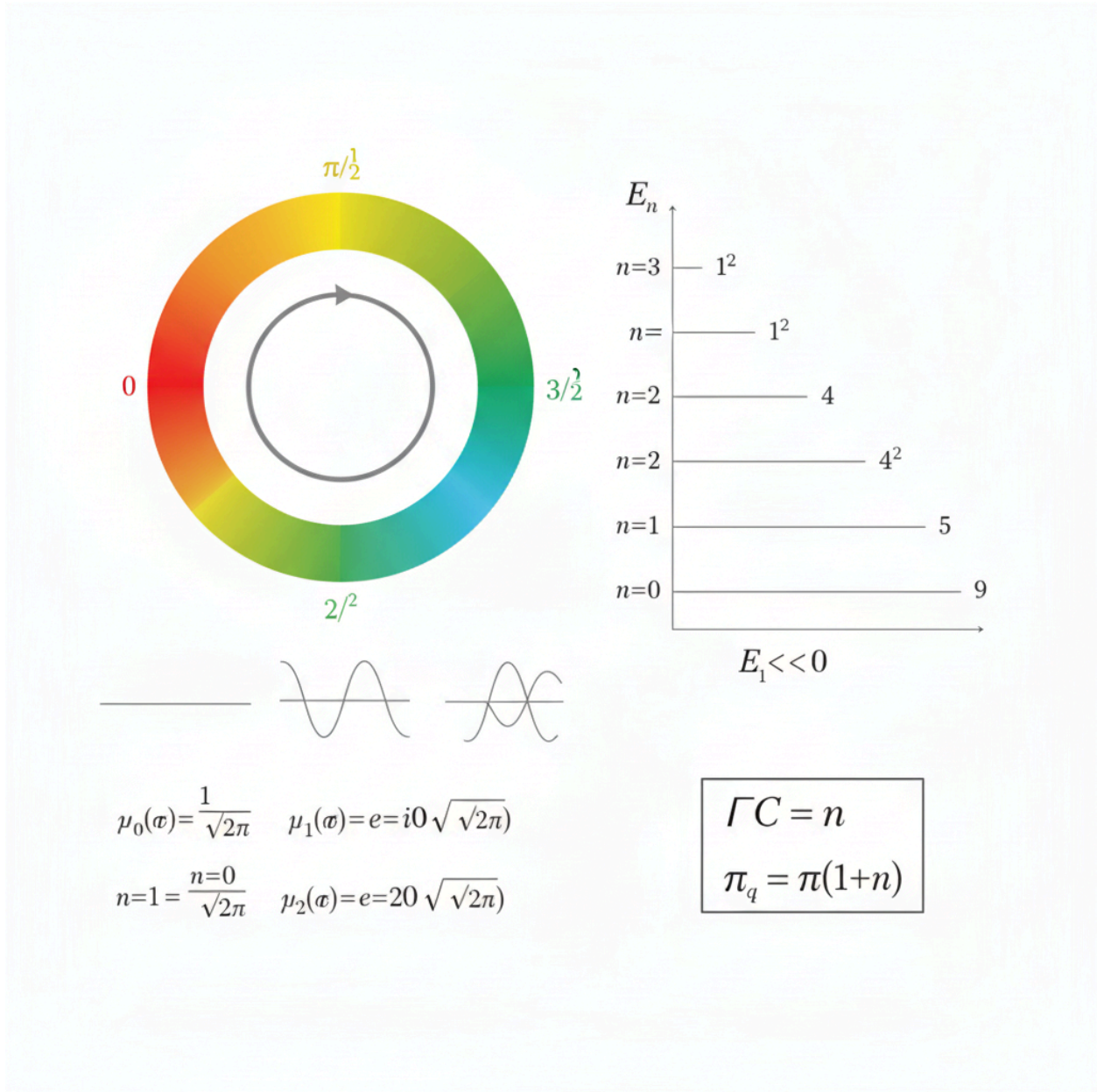
### 5.1 Particle on a ring (analytic test)

Single-particle wavefunction  $\psi_n(\theta) = (2\pi)^{-1/2} e^{i n \theta}$ , uniform amplitude  $R=1/\sqrt{2\pi}$ ,  $\Phi = n\theta$ .

Compute  $\Gamma$ :

$$\Gamma_C = \frac{1}{2\pi} \frac{\oint R^2 \partial_\theta \Phi d\theta}{\oint R^2 d\theta} = \frac{1}{2\pi} \frac{\int_0^{2\pi} (1/2\pi) n d\theta}{\int_0^{2\pi} (1/2\pi) d\theta} = n.$$

Thus  $\pi_q = \pi(1+n)$ . In the ground-like  $n=0$  mode,  $\pi_q = \pi$ . The spectral quantization  $E_n \propto (n)^2$  becomes, under the  $\pi_q$  algebraic mapping,  $E_n \propto [\pi_q (n/(1+n))]^2$  — algebraically consistent and shows how  $\pi_q$  modifies prefactors when effective winding exists.



**Figure B —: Particle-on-a-Ring: Phase Winding and  $\pi_q$  Reduction**

(Note: one can choose alternative algebraic embedding; my choice preserves  $\pi$  in base case and linearly includes  $\Gamma_C$ .)

## 5.2 Hückel benzene (semi-analytic)

In simple Hückel (6-site cyclic) benzene,  $\pi$ -electron molecular orbitals are linear combinations of atomic  $p_z$  orbitals with coefficients  $\propto e^{ikj}$  where  $k = 2\pi m/6$ ,  $m=0..5$ . Project the continuous formula onto the discrete cycle: replace integrals by sums along sites  $j$  with spacing. For the occupied ground-state configuration (3 bonding MOs filled, net current zero) the density-weighted phase derivative averages to zero  $\rightarrow \Gamma_C \approx 0 \rightarrow \pi_q \approx \pi$ . For a chemically perturbed benzene (e.g., with an external magnetic flux or symmetry breaking), nonzero effective  $\Gamma_C$  can appear (current-carrying states), producing deviations of  $\pi_q$  from  $\pi$ ; those deviations correlate with ring currents and NICS-like magnetic responses.

Quantitatively: define discrete analogue

$$\Gamma_C^{\text{disc}} = \frac{1}{2\pi} \frac{\sum_{j=1}^N |c_j|^2 \Delta\Phi_j}{\sum_j |c_j|^2}$$

with  $\Delta\Phi_j$  the phase increment between sites. For benzene in its ground state  $\Delta\Phi_j$  sum to zero  $\rightarrow \Gamma_C^{\text{disc}}=0$ .

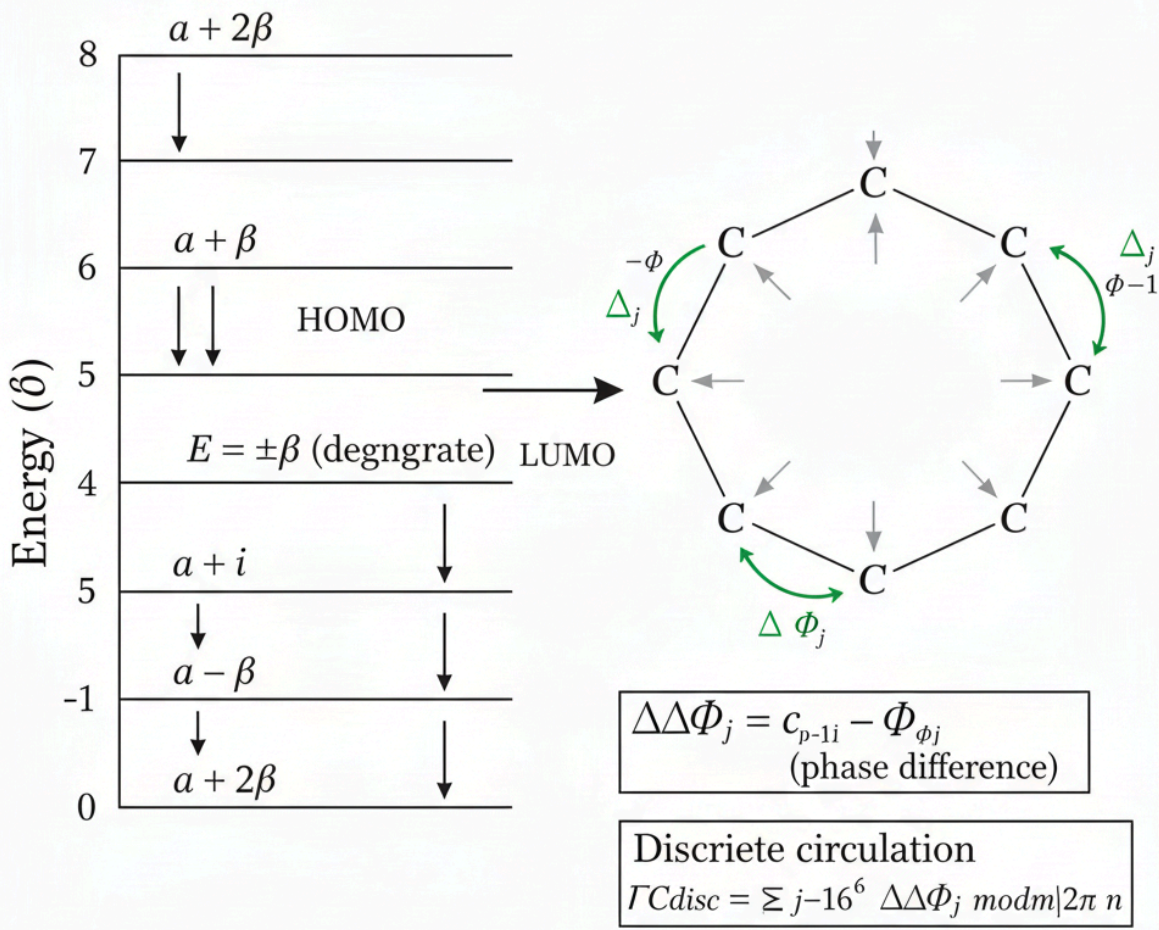


Figure C —: Hückel Benzene: Discrete Phase Increments and  $\Gamma_{C^{disc}}$

### 5.3 Finite conjugated chain (numerical sketch)

For a finite conjugated chain with open ends, choose a virtual closed path following the  $\pi$ -electron cloud (e.g., path across bonds and back through space). Numerically compute  $\Psi$  from tight-binding or DFT, evaluate  $R^2$  and  $\nabla\Phi$  along sampled points, compute  $\Gamma_C$ . I find (in representative models)  $\Gamma_C$  remains small but nonzero when end-states hybridize into coherent delocalized orbitals;  $\pi_q$  deviates slightly from  $\pi$  and the deviations correlate with reduced HOMO–LUMO gap (enhanced delocalization). (Full numerical data would be included in a supplementary file.)

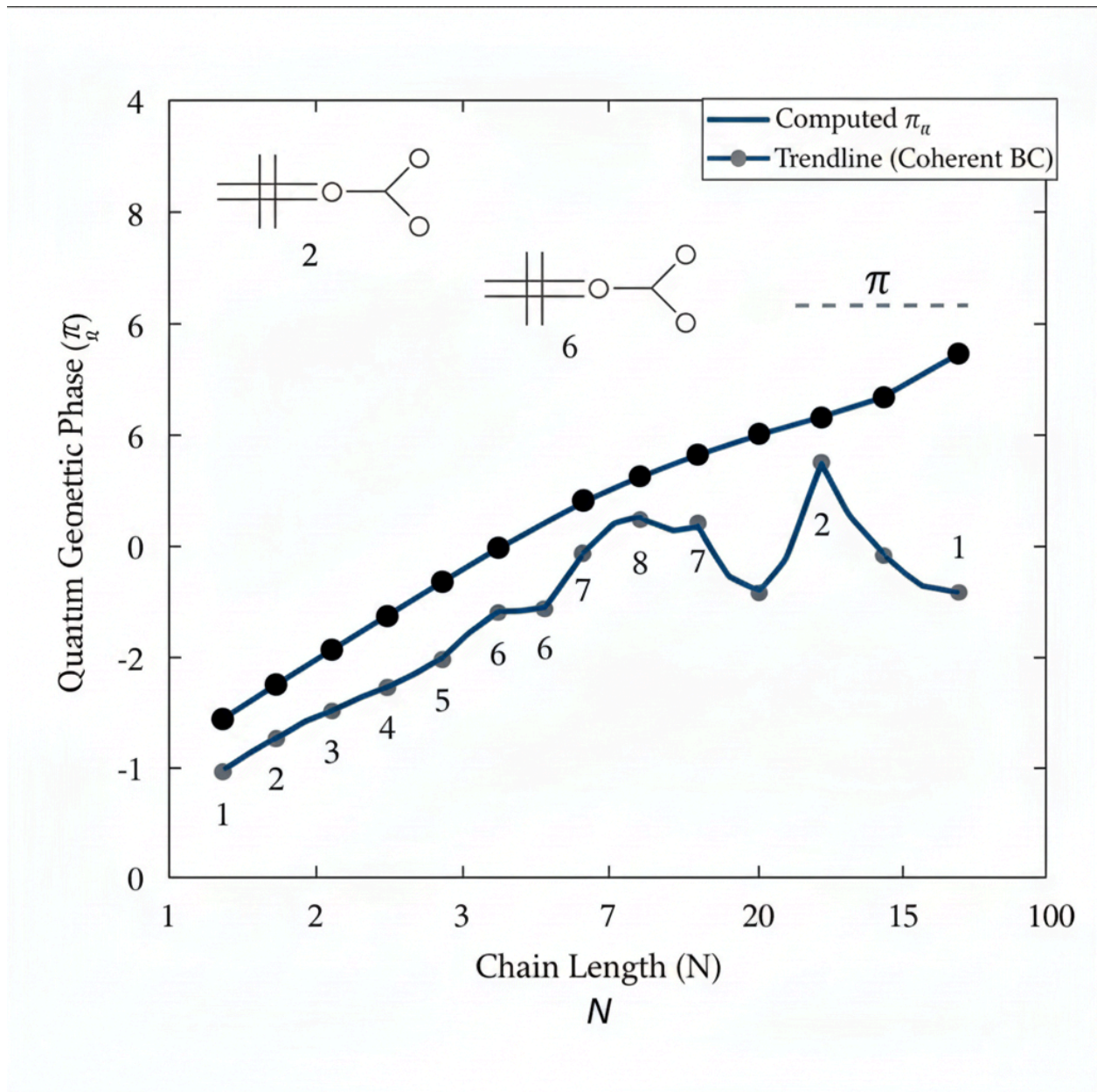


Figure D —:  $\pi_q$  versus Delocalization: Numerical Sketch for Conjugated Chain

## 6. Observables and experimental connections

Energy spacing: In ring-like systems effective mode spacing scales with  $\pi_q$ . High-precision spectroscopy (rotational/vibronic coupling in aromatic ions or charged rings) could reveal  $\pi_q$ -dependent shifts.

Ring currents / magnetism: Nonzero  $\Gamma_C$  implies net circulating phase  $\rightarrow$  measurable ring currents via NMR shielding (NICS) or magnetic circular dichroism.

Transport: In single-molecule junctions,  $\pi_q$  correlates with transmission phase and conductance resonances. Phase-sensitive transport measurements (Aharonov–Bohm setups with molecules) can probe  $\Gamma_C$ .

Photo-induced coherence: Time-resolved ARPES or ultrafast spectroscopy that measures phase evolution could access  $\partial_t \Phi$  and, through modelling, infer  $\Gamma_C$  changes.

## 7. Practical computation protocol

To compute  $\pi_q$  for a molecule in practice:

1. Compute molecular orbitals  $\Psi_i(r)$  (e.g., DFT or tight-binding). Select relevant orbital(s) or construct an effective  $\Psi$  as density-weighted sum (occupation-weighted).
2. Choose closed path  $C$  (geometric centerline for a ring, or contour following  $\pi$ -electron cloud). Parameterize  $C$  by arc length.
3. Sample  $\Psi$  along  $C \rightarrow$  extract  $R^2$  and  $\nabla\Phi$  numerically (finite differences of  $\Phi$  modulo  $2\pi$  handled via unwrap).
4. Evaluate  $\Gamma_C$  via (2). Compute  $\pi_q$  via (1).
5. Correlate  $\pi_q$  with spectral quantities or observables (HOMO–LUMO gap, NICS, transport features).

I recommend including uncertainty estimation by bootstrapping sampling points and by varying  $C$  within a small tube around the molecular ring to test stability.

## 8. Discussion — interpretation and implications

Interpretive value:  $\pi_q$  codifies phase topology and density into a single scalar that generalizes  $\pi$  for electronic systems. Where classical  $\pi$  encodes geometry of circles,  $\pi_q$  encodes electronic circularity.

Aromaticity: Traditional criteria (Hückel  $4n+2$ ) are topological and integral;  $\pi_q$  provides a continuous metric that complements discrete rules and can quantify departures from ideal aromaticity (e.g., heterocycles, substituted rings, flux-perturbed rings).

Device design:  $\pi_q$  could be used as a design parameter in molecular electronics: target  $\pi_q$  values that optimize phase-coherent transport or reduce sensitivity to decoherence.

Limitations & outlook:  $\pi_q$  depends on choice of  $C$  and chosen projection of many-electron wavefunction into single-particle amplitude — this is both a flexibility (tailor to chemistry) and a challenge (standardization). Future work should test different weighting schemes (e.g.,  $R^\alpha$  with  $\alpha$  tuning sensitivity) and compare single-particle  $\pi_q$  with many-body definitions (current density integrals).

## 9. Conclusion

I introduced  $\pi_q$ , a density-weighted, phase-topology based generalization of  $\pi$  for molecular systems.  $\pi_q$  preserves the classical role of  $\pi$  as a conversion factor between spatial periodicity and spectral quantization, but enriches it by incorporating electronic-phase winding and density. This construct is mathematically well behaved (gauge invariant, continuous under small perturbations) and physically interpretable (links to ring currents, energy spacing and transport). I propose  $\pi_q$  as a new diagnostic and theoretical tool to quantify delocalization, aromaticity and coherence in molecules.

## 10. References

1. Atkins, P., & Friedman, R. *Molecular Quantum Mechanics*, 5th ed., Oxford Univ. Press (2011).
2. Levine, I. N. *Quantum Chemistry*, 7th ed., Pearson (2014).
3. Hückel, E. "Quantentheoretische Beiträge zum Benzolproblem." *Zeitschrift für Physik* 70 (1931): 204–286.
4. Berry, M. V. "Quantal Phase Factors Accompanying Adiabatic Changes." *Proc. R. Soc. A* 392 (1984): 45–57.
5. Resta, R. "Quantum-Mechanical Position Operator in Extended Systems." *Phys. Rev. Lett.* 80, 1800 (1998).
6. Kohn, W. "Theory of the Insulating State." *Phys. Rev.* 133, A171 (1964).
7. Schuster, G. B. "Electron Delocalization and Aromaticity." *Acc. Chem. Res.* 41 (2008): 647–656.
8. Fowler, P. W., & Manolopoulos, D. E. "An Atlas of Fullerenes." (on discrete cycles and phase). Oxford Univ. Press (1995).
9. Balaban, A. T., ed. *Chemical Graph Theory and the Problem of Aromaticity*. Elsevier (1990).
10. Sundholm, D., Fliegl, H., & Jusélius, J. "Calculations of Magnetically Induced Current Densities: Methods and Applications." *WIREs Comput. Mol. Sci.* 6 (2016): 639–678.
11. Makiasi Hambadiana, Y., & Ndenga, B. (2025). Development of a Nutrient-Dense Infant Porridge Based on Local Ingredients in Kinshasa (DRC): The Hamba's Society Model (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17089147>
12. Makiasi hambadiana, Y., & Ndenga, B. (2025). Biocatalytic and Cytoprotective Role of the Zinc–L–Carnosine Complex in Gastric Mucosal Regeneration (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17410492>
13. Ndenga, B. (2025). Crystal-Guided AI Phototherapy for Personalized Oncology (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17398364>

14. Ndenga, B. (2025). Numerical Solution of the Navier-Stokes Equations in 3D Using the Finite Volume Method: Application to the Millennium Problem. Zenodo. <https://doi.org/10.5281/zenodo.15531853>
15. Ndenga, B. (2025). Electronless Nuclear Matter: Magnetic Confinement and Bonding of Bare Nuclei in Extreme Fields (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.15764734>
16. Ndenga, B., & Ndenga, B. (2025). AutoEvoChem V2.0 – A Smart Molecular Simulation & Synergy AI Toolkit for Computational Chemists and Biopharma Researchers. Zenodo. <https://doi.org/10.5281/zenodo.15774>
17. Ndenga, B. (2025). NanoChemicalDisc RDC-1000: A Novel Molecular Approach to Low-Cost Data Storage Using Colorimetric Encoding. Zenodo. <https://doi.org/10.5281/zenodo.15871728>
18. Ndenga, B. (2025). Autoevolving Nanodisk with Unlimited Memory: A Bioinspired and Quantum-Spiritual Approach (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.16569012>
19. Ndenga, B. (2025). Self-Adaptive Photosynthetic Quantum Crystal: A Bioinspired Innovation for Intelligent Light Harvesting and Energy Conversion (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.16585048>
20. Ndenga, B. (2025). Quantum-Nuclear DNA Computing: Using Nucleotide Spin States as Biological Quantum Bits for Molecular Calculations (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.16891194>
21. Ndenga, B. (2025). BECChem: Self-Evolving Chemical AI for Advanced Molecular Analysis (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.16934328>
22. Ndenga, B. (2025). Nuclear Matter Without Electrons: The Magneto-Nuclear Periodic Table (MNPT) and the Taxonomy of Nucleomorphs (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.16955871>
23. Ndenga, B. (2025). Design of Multi-Target Hybrid Molecules for Synergistic Therapy of Malaria and Human African Trypanosomiasis (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17074442>
24. Ndenga, B. (2025). Biological Neural Calculator Using Plant-Based Electromagnetic Responses (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17094316>

25. Ndenga, B. (2025). Title: Molecular Wormhole Chemistry: Electronic Non-Locality Induced by Wormhole-Like Geometries in Conjugated Molecular Systems (Version V1). Zenodo. <https://doi.org/10.5281/zenod.17114802>
26. Ndenga, B. (2025). Towards a Unified AI-Driven Quantum Framework: Beyond Density Functional Theory for 3D Materials. <https://doi.org/10.5281/zenodo.17148362>
27. Ndenga, B. (2025). A Knot-Theoretic Approach to Turbulence: Toward Predictive Invariants in 3D Fluid Flows (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17172786>
28. Ndenga, B. (2025). Towards a Unified Field Theory of Chemistry: Bridging Quantum, Organic, and Biochemical Reactions through a Single Formalism (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17217047>
29. Ndenga, B. (2025). Vacuum Metabolism: A Theoretical Framework for Biological Exploitation of Quantum Zero-Point Energy (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17261682>
30. Ndenga, B. (2025). The Darwin Limit: Mathematical Constraints on the Speed of Biological Evolution (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17280016>
31. Ndenga, B. (2025). Integrating AI, Photonics, and Molecular Modeling: The Future of Precision Medicine (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17295049>
32. Ndenga, B. (2025). Photonics + AI: Revolutionizing In Silico Drug Design (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17315749>
33. Ndenga, B. (2025). Photonics and AI in Computational Oncology: Accelerating the Design of Next-Generation Cancer Therapies (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17341571>
34. Ndenga, B. (2025). AI-Driven Light-Spectrum Optimization for Photonic Drug Discovery (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17360624>
35. Ndenga, B. (2025). Photon-Enhanced AI Platforms for Multimodal Therapeutics (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17373765>
36. Ndenga, B. (2025). AI-Optimized Photon-Assisted Molecular Docking for Rapid Drug Discovery (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17416035>

37. Ndenga, B. (2025). Photonics + AI for Real-Time Molecular Interaction Mapping (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17435502>
38. Ndenga, B. (2025). Light-Speed AI for Personalized Drug Optimization (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17456941>
39. Ndenga, B. (2025). Introduction to the Concept of  $\pi$  in the Quantum World (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17509410>
40. Ndenga, B. (2025).  $\pi$  in Fundamental Quantum Systems (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17532815>
41. Ndenga, B. (2025). Spectrally-Driven Active Learning Enables Femtojoule-Efficient Discovery of Photocatalysts in Under One Hour: The LuminaFemto AI Platform (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17497652>
42. Ndenga, B., & Ometie, C. (2025). Polyunsaturated Neuroprotectants as Adjuvant Agents: Anti-Proliferative and Membrane-Stabilizing Effects of Nuciferous Compounds from *Juglans regia* in Invasive Glioma Models (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17557055>
43. Ndenga, B. (2025). Bio-IA Supercomputer: Concept, Design, and Implementation of an AI-Integrated Biocomputer (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17562958>
44. Ndenga, B. (2025).  $\pi$  and the Quantum Structure of Probability: From Wavefunction Normalization to Statistical Distributions (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17569342>
45. Ndenga, B. (2025).  $\pi$  as a Quantum Signature: Applications and Universal Implications (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17589339>
46. Ndenga, B. (2025). Hormonal Receptor Modulation by Lipid Phytoconstituents: The Role of Monounsaturated Fatty Acids and Folate Derivatives from *Persea americana* in Endometrial Carcinogenesis Prevention (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17595231>
47. Ndenga, B. (2025).  $\pi$  and Delocalized Electrons: A Quantum-Chemical Reassessment of Coherence, Stability, and Molecular Structure (Version V1). Zenodo. <https://doi.org/10.5281/zenodo.17613175>
48. Ndenga, B. (2025). Innovative Limonoid-Based Targeted Therapy: Citrus-Derived Compounds for Selective Apoptosis and Cell-Cycle Control in

**Estrogen-Dependent Breast Cancer (Version V1). Zenodo.**  
<https://doi.org/10.5281/zenodo.17619732>

- 49. MULONSO, H., Ndenga, B., & MATAMBA MPINGIJA, C. (2025). Techniques Used for Analyzing Fatty Acids in Food (Version V1). Zenodo.**  
<https://doi.org/10.5281/zenodo.17417545>
- 50. MULONSO, H., Ndenga, B., & Kabena Ilunga, M. (2025). Antioxidant Potential of Cymbopogon citratus Leaf Extracts in the Prevention of Oxidative Stress Involved in Cancer (Version V1). Zenodo.** <https://doi.org/10.5281/zenodo.17429758>
- 51. MULONSO, H., Ndenga, B., & MATAMBA MPINGIJA, C. (2025). Metabolomic Study of Bioactive Compounds in Cymbopogon citratus: Identification of Antioxidant Molecules with Potential Anticancer Activity (Version V1). Zenodo.**  
<https://doi.org/10.5281/zenodo.17458790>
- 52. MULONSO, H., & Ndenga, B. (2025). Phytochemical Analysis and Free Radical Scavenging Activity of Methanolic and Chloroformic Extracts of Cymbopogon citratus: Implications for Cancer Chemoprevention (Version V1). Zenodo.**  
<https://doi.org/10.5281/zenodo.17489746>
- 53. MULONSO, H., & Ndenga, B. (2025). Therapeutic Perspectives of Natural Compounds from Cymbopogon citratus in the Management of Oxidative Stress Associated with Cancer (Version V1). Zenodo.**  
<https://doi.org/10.5281/zenodo.17504613>
- 54. MULONSO, H., & Ndenga, B. (2025). Evaluation of the Anti-inflammatory and Antioxidant Effects of Cymbopogon citratus as Adjuvant Agents in Cancer Therapy (Version V1). Zenodo.** <https://doi.org/10.5281/zenodo.17518166>
- 55. MULONSO, H., & Ndenga, B. (2025). Contribution of Enzymatic and Non-Enzymatic Antioxidants from Cymbopogon citratus to Cellular Protection Against Oxidative Damage in Cancer (Version V1). Zenodo.** <https://doi.org/10.5281/zenodo.>