Optimal Trading Portfolio Allocation Enhancement with Maximum Drawdown Using Triple Penance Rule

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Abstract

This study focuses on optimal portfolio trading allocation, using historic trading data to propose an optimal allocation with risk consideration. The risk factors used sequentially in this study are Sharp Ratio (SR) and Maximum Drawdown (MDD). We use the Sequential Least Squares programming (SLSQP) solver to minimise the negative adapted Sharp Ratio for trading profit and loss (PNL) and then we use the Triple Penance Rule assumption for modelling a mathematical formula to represent the relationship between the MDD and the Time under Water (TUW). In this paper, we introduce the risk measure driven by the MDD as the area of the curve during the TUW. The inverse of this area is used to define a proportional weight to the MDD value. The final weight is obtained by applying an algorithm for removing all MDD outliers in the dataset after SLSQP optimization. The final optimal weight is a combination of the calculated proportional PNL weights and the weights from the MDD area. The benchmark is the equal weight portfolio and the maximal return portfolio from Monte Carlo Simulation (MCS). The tested portfolios are two portfolios made from stocks and securities of the Dow Jones Industrial Average (DOJI) and the NASDAQ Composite (COMP). The initial selection is made from only securities that offer at least five years of historical prices. The results show a good

compromise in weight allocation between maximising the PNL and minimising MDD and Sharp ratio.

Keywords: Sharp Ratio, Profit and Loss, Time under Water, Maximum Drawdown, Sequential Least Squares Programming, Triple Penance Rule, Monte Carlo Simulation

1. Introduction

Portfolio allocation has always been a serious matter for all parties involved in portfolio management. In general, investors like to be risk averse, but in the real world, risk is always present. Seeking a compromise between high profit and low risk leads to optimization. In this paper, we work on a trading portfolio, which is a portfolio built only by applying a trading strategy over all assets in the portfolio. The same strategy can perform differently on each security depending on the volatility. In the study of portfolio sensitivity, Stoyanov S. et al. (2013) define value-at-risk (VaR) as stating that VaR calculates a portfolio's overall possible loss from market risk with a given degree of confidence over a specified time interval. VaR has the major disadvantage that it cannot handle diversification because of its inability to be sub-additive. E. Fisher et al. (2010) [2] discuss another risk metric, conditional value-at-risk (CVaR), also known as the estimated shortfall. CVaR is obtained by a weighted average of the extreme losses in the tail of the potential return distribution, beyond the tail loss point at risk value (VaR). However, when it comes to rebalancing and optimising returns, these two metrics are incapable of handling portfolio allocation. Other risk measures like volatility are used by portfolio managers (PM) in the form of Sharp ratio analysis. The Sharp ratio is the annualised return divided by the volatility. Then, an optimal portfolio is obtained by maximising the Sharp ratio, which means minimising the volatility, therefore the risk, and maximising the return. Optimization is then achieved with the compromise of a good return level with diversification. This common and widely used approach is more accurate for holding a portfolio. In the case of a trading portfolio, the trading strategy plays a huge role by driving the possibility to manage losses with drawdown. The Maximum Drawdown (MDD) calculates the maximum decline in the value of the investment. It is characterised by the difference between the value of the lowest trough and the highest pre-trough peak. When the value of an asset or an investment has gone through many boom-bust cycles, MDD is assessed over a long period. The MDD is indeed a downside risk measure over a given period of time. Investors also evaluate the risk driven by the MDD in terms of time under water (TUW) and decide on a stop-out accordingly to whether the TUW is too long or not. Bailey, D. et al. (2012) [3] proposed an empirical framework for drawdown (DD) and TUW. The TUW increases with the DD in a serial correlation, so portfolio managers need to manage the DD as a whole factor independently of the Sharp ratio. Thus, the resulting model, called the Triple Penance Rule. It proceeds at a given confidence level to determine the MDD and the maximum TUW. Lopez de Prado (2013) [4] with the triple penance rule, theoretically estimates the recovery time using the triple penance rule assumption. The rule states that the recovering time is three times the time taken to realise an MDD.

In this paper, we focus on a trading portfolio instead of a holding one. There is a lot of work out there on holding portfolio optimization. The return is used to calculate the volatility, and thus, the risk. For a trading portfolio, the return is calculated in terms of profit and loss (PNL). One of the biggest challenges in a trading portfolio is to make a good allocation and rebalance in order to maximise the profit. Portfolio performance depends on the trading strategy, and with that strategy, traders can control the loss, the volatility, and the drawdown.

The main goal of this paper is to develop a method and a framework for optimal trading portfolio allocation that sequentially uses an adapted Sharp ratio after a first optimization using the numerical method Least Square explained by Nocedal, J. et al. (2000) [5]. We use the numerical implementation of the least square method as a Sequential Least Squares Programming (SLSQP) optimization using the Python language implemented by the Scipy library [6]. We then select our proposed algorithm using the MDD with triple penance rule assumption to evaluate a risk metric used to determine the weights accordingly. The rest of the paper is sectioned as follows:

Part 2 defines the adapted Sharp ratio for the trading portfolio, Part 3 is an empirical mathematical model of the MDD based on the triple penance rule assumption, Part 4 is the framework process, and Part 5 is the numerical results of the framework's simulation.

2. Adapted Sharp ratio for a trading portfolio formulation

Buy assumption the trading strategy is given thus we focus on PNL as a return measure. By assumption in this study all position are long thus the strategy only buy and sell.

2.1. Problem Formulation

The rate of return the PNL $\lambda_t^{(a)}$of an asset *a* in this scope can be calculated for a single period by subtracting the Buy price value at the period *t-1* $B_{t-1}^{(a)}$ with the Sell price value at the period t $S_t^{(a)}$ dividing by the asset Buy price value at period *t-1*.

$$\lambda_t^{(a)} = \frac{S_t^{(a)}}{B_{t-1}^{(a)}} - 1 \tag{1}$$

Portfolio Ψ of N asset PNL adapted Rate of Return is expressed as follow:

$$\lambda_N^{(\Psi)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} w^{(a)} \lambda_{T^{(a)}}^{(a)} \tag{2}$$

Where N is to total number of asset and $T^{(a)}$ is the total number of trades for an asset a. From the equation (2) we replace the PNL return rate by the expression as follow:

$$\lambda_N^{(\Psi)} = \sum_{\boldsymbol{a=1}}^{\boldsymbol{N}} \sum_{k=1}^{T^{(a)}} w^{(a)} \left(\frac{S_k^{(a)}}{B_{k-1}^{(a)}} - 1 \right) \tag{3}$$

Where k is the current trade with$k \in \{1,2 \dots\dots \mathrm{T}^{(a)}\}$ and $a \in \{1,2 \dots\dots N\}$. $w^{(a)}$is the weight of the asset a.

By principle there is not constant frequency of returns when trade. In addition, the return is not annualized thus we consider the adapted volatility as the sum of the variance of the returns. For a portfolio of N assets, the adapted volatility $V^{(\Psi)}$ is expressed as:

$$\mathrm{V}_N^{(\Psi)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \mathrm{Var}_i^{(\Psi)} \tag{4}$$

$$\mathrm{Var}_N^{(\Psi)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \sum_{j=1}^{N} w^{(i)} w^{(j)} \, Cov\left(\lambda_{T^{(i)}}^{(i)}, \lambda_{T^{(j)}}^{(j)} \right) \tag{5}$$

The portfolio adapted volatility is:

$$\mathrm{V}_N^{(\Psi)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \left(\sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \sum_{j=1}^{N} w^{(i)} w^{(j)} \, Cov\left(\lambda_{T^{(i)}}^{(i)}, \lambda_{T^{(j)}}^{(j)} \right) \right) \tag{6}$$

The adapted Sharp ratio $H^{(\Psi)}$for a portfolio of N asset is then expressed as follow:

$$\mathrm{H}_N^{(\Psi)} = \frac{\lambda_N^{(\Psi)}}{\mathrm{V}_N^{(\Psi)}} \quad (7)$$

When replacing with equation (3) and (5)

$$\mathrm{H}_N^{(\Psi)} = \frac{\sum_{a=1}^{N}\sum_{k=1}^{T^{(a)}} w^{(a)}\left(\frac{S_k^{(a)}}{B_{k-1}^{(a)}}-1\right)}{\sum_{i=1}^{N}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w^{(i)}w^{(j)}Cov\left(\lambda_{T^{(i)}}^{(i)},\lambda_{T^{(j)}}^{(j)}\right)\right)} \quad (8)$$

Mikkel Rasmussen (2003) discusses traditional formulation or rate of return and volatility for holding portfolio [6].

2.2. Data Application Portfolio Optimization using SLSQP solver

The dataset is composed of two sets of stock assets from the Dow Jones Industrial Average (DOJI) and a set of securities from the Nasdaq Composite (COMP). Only securities that offer at least 5 years of historical data are selected.

Table 1: Set of assets for Studied Portfolio

DOJI Assets	COMP Assets
MMM , AXP , AMGN , AAPL , BA , CVX , CSCO , KO , GS , HD , HON , INTC , IBM , JNJ ,JPM , MCD , MRK , MSFT , NKE , PG , CRM , TRV , UNH , VZ , V , WBA , DIS	TWOU, FLWS, SRCE, FCCY, VNET

We simulate back testing with a given simple trading strategy (the strategy is not studied in this paper). See the definition of that strategy in Appendix 1. The period used is five years of historical data. We also calculate the holding optimization as a benchmark and the Monte Carlo simulation (MCS). The tables below show the computed PNL without weights as well as the computed weights from MCS and SLSQP for both the DOJI and COMP portfolios.

Table 2: Weights after back test and optimization with SLSQP and MCS For COMP

Tickers	Total PNL	MDD	Equal Weights	Weights MCS	Weights SLSQP	Weights SLSQP Holding
TWOU	0.79364669	49.1361	0.2	0.4114565	0.1744	0.0000

FLWS	0.84765964	31.2623	0.2	0.46005341	0.4174	0.5527
SRCE	-0.38342171	56.4063	0.2	0.00797417	0	0.0000
FCCY	0.31074292	30.0643	0.2	0.11275217	0.3687	0.3417
VNET	0.17643936	75.2389	0.2	0.00776375	0.0394	0.1056

The total PNL is calculated without allocation after running the back test. The MCS picked is the one that, in this simulation, offered the maximum PNL return. As we can see, negative PNL has no weights after optimization. Holding a daily optimal allocation is also given here. The following table shows the same results but for DOJI.

Table 3: Weights after back test and optimization with SLSQP and MCS For DOJI

Tickers	**Total PNL**	**MDD**	**Equal Weights**	**Weights MCS**	**Weights SLSQP**	**Weights SLSQP Holding**
MMM	-0.3845	54.1138	0.037	0.0022	0	0
AXP	0.5725	30.5607	0.037	0.0032	0.0681	0
AMGN	0.0266	25.0107	0.037	0.0390	0.0092	0
AAPL	0.4873	74.9085	0.037	0.0130	0.0214	0
BA	1.1347	43.777	0.037	0.0763	0.0131	0
CVX	-0.2444	39.166	0.037	0.0162	0	0
CSCO	0.1852	28.7472	0.037	0.0009	0.0261	0
KO	-0.1369	21.5938	0.037	0.0395	0	0
GS	0.7516	34.1927	0.037	0.0633	0.0286	0
HD	0.6593	19.935	0.037	0.0234	0.0894	0
HON	0.4163	27.3082	0.037	0.0427	0.0906	0
INTC	-0.0346	38.4726	0.037	0.0096	0	0
IBM	-0.3403	44.8267	0.037	0.0039	0	0
JNJ	-0.0100	21.7925	0.037	0.0570	0	0
JPM	0.5968	35.1283	0.037	0.0138	0.0426	0
MCD	0.3924	19.7727	0.037	0.0693	0.0575	0
MRK	-0.1963	34.9886	0.037	0.0648	0	0
MSFT	0.9152	26.4209	0.037	0.0761	0.1062	0.7588
NKE	0.6124	23.8248	0.037	0.0882	0.0671	0.1234
PG	0.2304	20.1388	0.037	0.0055	0.0863	0
CRM	0.6431	27.889	0.037	0.0424	0.0534	0
TRV	0.2274	29.073	0.037	0.0007	0.0322	0
UNH	0.3058	27.5439	0.037	0.0572	0.054	0.1178
VZ	-0.3130	33.3535	0.037	0.0015	0	0
V	0.5690	19.2203	0.037	0.0888	0.083	0

WBA	-0.3259	42.6745	0.037	0.0116	0	0
DIS	0.5417	20.0683	0.037	0.0897	0.0713	0

From these two portfolios above, we can observe a great convergence with the optimizer that affected 0 weight for for all negative PNL and the same asset corresponding in holding are also affected with 0.

The PNL results compared to others methods is done in the table below.

Table 4: Comparative Total PNL of Optimized Portfolios Vs MCS maximum PNL portfolio

Portfolio	**Total PNL MCS %**	**Total PNL SLSQP %**	**Total PNL Equal Weight %**	**Total PNL SLSQP Holding %**
DOJI	46.43	53.76	26.94	80.61
COMP	75.43	61.38	34.90	59.33

Optimized trading portfolios with SLSQP show better results than equal-weighted portfolios. Due to the lack of risk handling, MCS Total PNL can sometimes perform better than the optimised. Therefore, optimization with an adapted Sharp ratio can provide a reliable performance result, and the returns are close to those of MCS performance. The problem is the risk evaluation. In this case, it considers the adapted volatility but not the MDD, which is one of the crucial measurements of risk for a trading strategy.

That is why in the next section we develop in extension a new framework optimization algorithm that minimises the average MDD while maximising the PNL with the MDD new risk measure proposal.

3. Maximum drawdown mathematical expression using tripe penance rule approximation

We propose an empirical mathematical formula to estimate the behaviour of the MDD based on the tripe penance rule. The goal is to provide an estimation of the time under water with the assumption that the MDD follows the triple penance rule.

$$\emptyset^{(i)}(t) = (-et - e)e^{-t} + \alpha^{(i)}e^{1-t} + \frac{\beta^{(i)}t^2}{2} - \alpha^{(i)}\beta^{(i)}t + K^{(i)} \quad (9)$$

Where $\emptyset^{(i)}(t)$ the maximum drawdown function of t for a given the time, e is the exponential constant $\alpha^{(i)}$, $\beta^{(i)}$ and $K^{(i)}$ Each asset has constant variables, such as We need to determine the values of the constant according to the triple penance rule assumption for this model, listed as follows:

1- The beginning of the drawdown which is the peak is the point of coordinate (0,0)

2- The period of MDD (τ) occurs at the minimum of the function at the MDD (γ)

3- It takes 4 times the period of MDD (τ) to recover at the 0 level at the coordinate (4τ,0)

Expression of the derivative from equation (8) is:

$$\frac{\partial \emptyset^{(i)}(t)}{\partial t} = \left(t - \alpha^{(i)}\right)\left(e^{1-t} + \beta^{(i)}\right) \tag{10}$$

To determine the constants for each asset we apply the following conditions:

$$\frac{\partial \emptyset^{(i)}(t)}{\partial t | t = \tau} = 0$$

$$\emptyset^{(i)}(t)_{|t=0} = 0$$

$$\emptyset^{(i)}(t)_{|t=\tau} = \gamma$$

$$\emptyset^{(i)}(t)_{|t=4\tau} = 0$$

This system of equations is solved numerically, we used Python as the programming language to do so. The following figure is the plot simulation of the MDD function described above for three values of MDD of -5, -15, and -25. The MDD is a negative value. These equations are solved numerically for a given value of MDD. See the figure below.

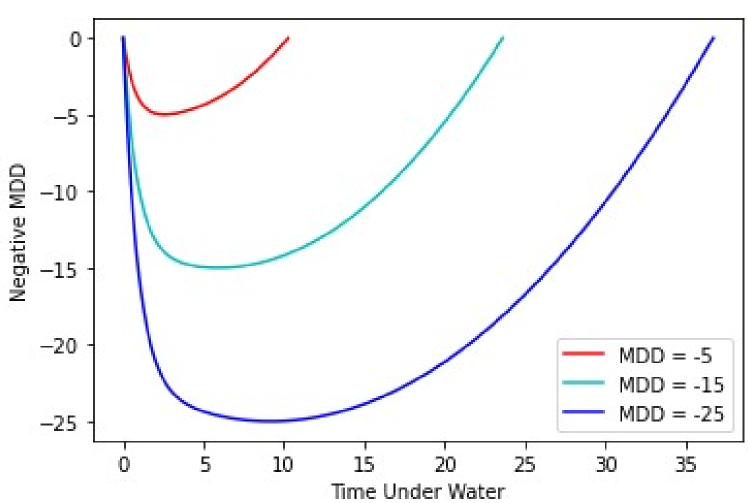


Fig 1: Simulation of the Mathematical model of MDD using triple penance approach

The time to realise MDD from this simulation increases with the MDD and the time under water as well. Using it for risk estimation is the same as using it for one of the most difficult challenges for traders: minimising their MDD due to recovery time. and the risk of liquidation.

4. Theory of the risk evaluation using MDD

The value of MDD itself does not give information about the time under water. That is why in this study we want to add the effect of the time under water to establish a risk assessment.

4.1. Assumption

- The MDD can be approximated with the Triple Penance Rule with the formula proposed
- The measure of the area under water includes the whole time under water (Tuw)
- The area increases with the MDD following the formula proposed above (9)
- The MDD absolute decrease with the inverse of the area under water
- The area or it inverse better represent the risk that the only MDD

4.2.Area under water estimation

The area under water is calculated by the double integral of the calculated for each asset (i) as follows:

$$A_{uw}^{(i)} = \int_{-M^{(i)}}^{0} \int_{0}^{4\tau^{(i)}} \left((-et - e)e^{-t} + \alpha^{(i)} e^{1-t} + \frac{\beta^{(i)} t^2}{2} - \alpha^{(i)} \beta^{(i)} t + K^{(i)} \right) dt d\emptyset^{(i)} \quad (11)$$

Where A_{uw} is the area under water for each security, M is the MDD value.

The inverse value of A_{uw} is chosen to evaluate the normalised weight of each security in the portfolio. By implementing this approach, we make sure that the most weighted area is the lowest area, thus the lowest MDD. It implies that the least risky in the dataset is the one with the largest area, thus the largest MDD, and thus the riskiest in the portfolio.

To minimise the risk in this scope, the logic is to minimise the MDD values of securities and maximise their weight in the portfolio. However, the optimization goal is not just risk minimization but also a compromise between minimum risk and maximum return.

Let define some metric variables created.

- The PNL proportional weight w_p is defined by the PNL of the security divided by some of the PNL of the portfolio. Given by the formula below,

$$w_p{}^{(i)} = \frac{\lambda_{T^{(i)}}^{(i)}}{\sum_{i=1}^{N} \lambda_{T^{(i)}}^{(i)}} \tag{12}$$

- The MDD inverse of Area under water proportional weight w_m

$$w_m^{(i)} = \frac{\left(A_{uw}^{(i)}\right)^{-1}}{\sum_{i=1}^{N}\left(A_{uw}^{(i)}\right)^{-1}} \tag{13}$$

$$w_m^{(i)} = \frac{1}{A_{uw}^{(i)}\left(\sum_{i=1}^{N}\left(A_{uw}^{(i)}\right)^{-1}\right)} \tag{14}$$

The relationship between the MDD and the inverse area is shown in the figure below, where we can see the value of the MDD decreasing with the value of the inverse area.

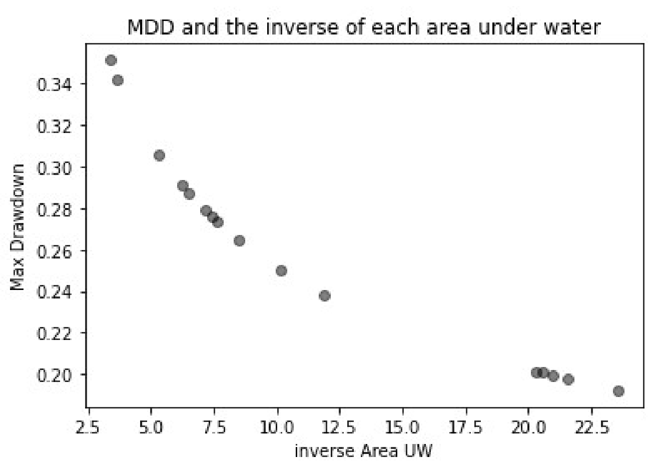


Figure 2: Maximum Drawdown and the inverse of the area Under Water for DOJI

The shaped curve in this figure represents the risk curve that we use to determine the best option for the weight.

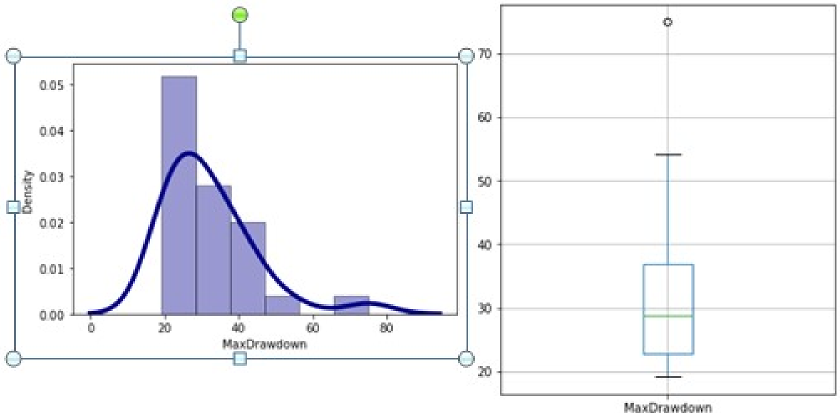
5. Design of the framework algorithm to select optimal allocation

The framework to select and optimise portfolio allocation is based on the top of the one got from the solver, SLSPQ, and follows these major steps.

- Get historical data of the initial set of assets in the portfolio and back test it for each security to get the PNL for each trade.
- Calculate the adapted Sharp ratio, then run the SLSQP solver to minimise the negative adapted Sharp ratio and output the optimal results.
- Recursively remove the MDD outlier point until you converge to a set without outliers.
- Calculate the inverse of being under water as the weights are related and also the proportional PNL weights
- Determine the optimal weight within the max, min, and average between the proportional PNL weights and the weights from the MDD inverse area of the time under water.
- Calculate the optimal allocation by reducing the sum of all weights to one, and then calculate the cumulative PNL for each asset. Calculate other cumulative PNLs like the one from only MDD area time under water weights.
- Calculate the different total PNLs.

5.1.Determination of outliers

We want to minimise the MDD, so we remove the outliers, which correspond to any MDD greater than the maximum number given by the box plot.



	MaxDrawdown
count	27
mean	32.01859
std	12.47559
min	19.2203
25%	22.80865
50%	28.7472
75%	36.80045
max	74.9085

Figure 3: MDD statistic for DOJI Initial Set of asset

From the statistics in the figure above, we can see an outlier at the value of 74.9, which is greater than the maximum number. This outlier is removed and a new set is evaluated. If there is a new outlier, this one is also removed until we have a set without an outlier.

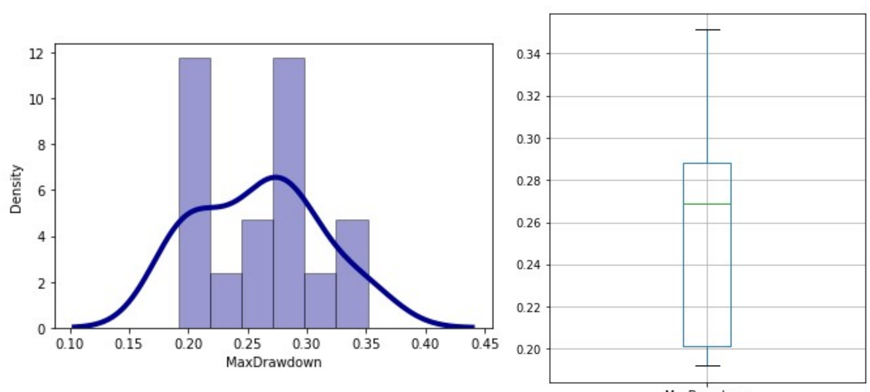
The condition of defining a point as an outlier is:

$$MDD^{(k)} \geq Maximum \quad (15)$$

$$Maximum = Q_{75} + 1.5 \times (Q_{75} - Q_{25}) \quad (16)$$

Where Q_{75} and Q_{25} are quartiles respectively 75th and the 25th.

The goal of this selection is to have a distribution more close to normal and reject all high-risk conditions due to high MDD. After removing outliers after SLSQP optimization, the dataset looks like the following:



	MaxDrawdown100
count	16.0
mean	25.9271
std	5.1170
min	19.2203
25%	20.1211
50%	26.8645
75%	28.8286
max	35.1283

Figure 4: MDD statistic for DOJI after applying SLSQP and remove outliers

Here we can see the data is closer to the normal distribution and, obviously, the standard deviation is reduced. So, for our framework, this is the optimal set of assets to compute allocation.

5.2.Algorithm pseudo code for optimal allocation selection

The algorithm pseudo code used is written below.

```
Begin Procedure
    input
        var List<String> tickers
        var Date dateStart
        var Date dateEnd
    getData(input)
        return List<Object> prices
    Foreach String price.ticker in prices:
        applyTradingStrategy(ticker) -> add result to list of tickers
    output file
        List<Number> profitAndLoss
        List<Number> maxDrawdowns
        List<String> tickers
    End Procedure
    Begin Optimization Procedure
    input file
        List<Number> profitAndLoss
        List<Number> maxDrawdowns
        List<String> tickers
    var volatility ← getSharpRatio(param: profitAndLoss)
```

```
var SharpRatio ← getSharpRatio(param: profitAndLoss)
var Object results  ← SLSQP_minimize(param: SharpRatio)
writeFile(results)
var List<String> newTickers ← getSLSQPPortfolio(results)
var List<Object> listMddAndPnl ← selectMDD(param: newTickers)
var Number Q1_25th ← maxDrawdowns.quantile(0.25)
var Number Q3_75th ← maxDrawdowns.quantile(0.75)
var Number IQR = Q3_75th - Q1_25t

var  Array<Number> outlier  ←  when  maxDrawdowns> Q3_75th + 1.5*IQR
While listMddAndPnl.maxDrawdownshas  outlier:
       listMddAndPnl.maxDrawdowns.remove(param: outlier)
var List<Number> invAuws
var List<Number> weightAuws
var List<Number> weightPnls
var List<Number> newWeights
var List<Number> tempWeights
Foreach Number maxDrawdown in listMddAndPnl.maxDrawdowns:
       var AreaUnderWater ←  integral(param maxDrawdown)
       var invAreaUnderWater = 1/AreaUnderWater
       invAuws.add(invAreaUnderWater)
Foreach Number invAreaUnderWater in invAuws:
       var weightAuw = invAreaUnderWater/invAuws.sum()
       weightAuws.add(weightAuw)
Foreach Number pnl in invAuws:
       var weighPnl = pnl/listMddAndPnl.ProfitLoss.sum()
       weightPnls.add(weighPnl)
var Number  medianWeightPnls = weightPnls.median()
var Number medianWeightAuws = weightAuws.median()
Case
       when  weightPnls.weighPnl >=  medianWeightPnls AND
weightPnls.weightAuw>=  medianWeightAuws:
```

```
        then tempWeights.add(max(weightPnls.weighPnl , weightPnls.weightAuw)):

        when weightPnls.weighPnl >= medianWeightPnls AND
        weightPnls.weightAuw<medianWeightAuws

        then tempWeights.add(mean(weightPnls.weighPnl, weightPnls.weightAuw))

        when weightPnls.weighPnl<medianWeightPnls AND  weightPnls.weightAuw>=
    medianWeightAuws

              then tempWeights.add(mean(weightPnls.weighPnl ,
    weightPnls.weightAuw))

        when weightPnls.weighPnl<medianWeightPnls AND
    weightPnls.weightAuw<medianWeightAuws

        then tempWeights.add(min(weightPnls.weighPnl , weightPnls.weightAuw))

    End

    Foreach Number tempWeight in tempWeights:

              var newWeight = tempWeight/tempWeights.sum()

              newWeights.add(newWeight)

    Output newWeights

End Procedure
```

By applying the method, we can calculate the optimal weight considering the MDD. Other weights are calculated as the MDD Area Weight, which is the weight only from the inverse of the area of the time under water. These values are completely independent of the Sharp ratio and concentrate solely on the risk posed by the MDD. The PNL weight is calculated proportionally to the cumulated PNL, so the higher the PNL, the higher the weight. The lowest PNL is the lowest weight. Optimal weights are the ones calculated with the algorithm described that combine the MDD area weight and the PNL weight.

Table 5: Different weights Calculated and the optimal for DOJI

Tickers	MDD	Equal Weights	MDD Area Weights	PNL Weights	Optimal Weights
AXP	30.56	0.0625	0.028739	0.074883	0.048381
AMGN	25.01	0.0625	0.054939	0.00348	0.027276
CSCO	28.75	0.0625	0.035049	0.024218	0.022614
GS	34.19	0.0625	0.019927	0.098303	0.055201
HD	19.94	0.0625	0.113554	0.086236	0.106036
HON	27.31	0.0625	0.041384	0.054445	0.038644
JPM	35.13	0.0625	0.018241	0.078054	0.04496

MCD	19.77	0.0625	0.11655	0.051324	0.07838
MSFT	26.42	0.0625	0.04604	0.119704	0.111779
NKE	23.82	0.0625	0.06422	0.080102	0.074799
PG	20.14	0.0625	0.109933	0.030131	0.065396
CRM	27.89	0.0625	0.038662	0.084108	0.057321
TRV	29.07	0.0625	0.033792	0.029741	0.027772
UNH	27.54	0.0625	0.04025	0.04	0.037352
V	19.22	0.0625	0.12755	0.074422	0.119106
DIS	20.07	0.0625	0.111169	0.070848	0.084983

Table 6: Different weights Calculated and the optimal for COMP

Tickers	MDD	Equal Weights	MDD Area Weights	PNL Weights	Optimal Weights
TWOU	49.14	0.25	0.0931	0.3729	0.2384
FLWS	31.26	0.25	0.4145	0.3982	0.4241
FCCY	30.06	0.25	0.4707	0.1460	0.3154
VNET	75.24	0.25	0.0216	0.0829	0.0221

From these weights, we can deduce the total PNL for each allocation and compare see tables below.

Table 7: Total PNL with different allocation

Portfolio	Total PNL Equal Weights	Total PNL Only PNL weights	Total PNL Only MDD Area weights	Total PNL Optimal Weights
DOJI	47.78	58.69 %	47.25%	**54.93 %**
COMP	53.21	69.35%	57.54 %	**65.06 %**

The SLSQP trading portfolio allocation in tables 5 and 6 shows the portfolio with only PNL proportional weights to the performance. They have in common that they do not include MDD in calculus as a risk measure parameter. The first one includes the Sharp ratio and the second just the performance.

The total PNL with only SLSQP weights obtained after proportional reduction gives the higher return, but also drives the higher risk. It does not take a Sharp ration or MDD as the risk criterion before evaluation. This portfolio can be proposed as a choice for investors willing to take more risk at the price of a better return.

Allocation with only weights deducted from MDD showed less performance than others. However, it is a conservative portfolio allocation with well-managed risk. This portfolio can be proposed to investors as a choice for those willing to take less risk while still being profitable anyway.

The last portfolio deducted from our algorithm is the one with a compromise between performance and MDD risk consideration. Its results showed satisfactory performance with great diversification of assets, not too far from the one from the SLSQP optimization solver. The final optimised portfolio always performs better than the equal allocation portfolio.

6. Conclusion

In this paper, we proposed an empirical mathematical model of MDD based on the triple penance rule assumption. This model allows us to evaluate the time under water for a given trading strategy. Hence, we proposed a new measure of risk which is determined by the integral of the MDD function between the beginning moment of the drawdown and the ending moment when the asset under trade has recovered its PNL. The optimum allocation is calculated by an algorithm that first performs an optimization with the SLSQP solver of the adapted Sharp ratio calculated with PNL realised during the whole historical period. Then, we removed all outliers. The inverse of the area under water is then used to evaluate weights only proper for MDD. The value of this metric allowed us to quote the weight rank accordingly to the value of the area with the logic that implies the risk. Hence, the decreasing values of the inverse of the area involve increasing risk, so weight decreases. These weights, deduced from MDD, were combined with the proportional PNL weights to produce the final optimal allocation. The results showed a better performance than the equal-weighted portfolio. Three types of portfolios can be output to give more flexibility of choice to investors. With this developed framework, a rebalancing strategy can be carried out each time as needed.

Appendix

1. Trading Strategy

- Step 1: Get the trend.
    - Slope of the EMA is positive (Up)
    - The last Heiken-Ashi Bar is blue (the Heiken-Ashi close price is greater than the open price)
    - The price is above the EMA.
    - CCI Cross above the zero line
- Step 2: Trigger Buy Entry Signal when 12-period EMA crosses above 26-period EMA, which also means MACD crosses above the zero line
- Step 3: Exit signal when 12-period EMA crosses bellow 26-period EMA, which mean MACD crosses bellow the zero line

2. Algorithm Flow Diagram

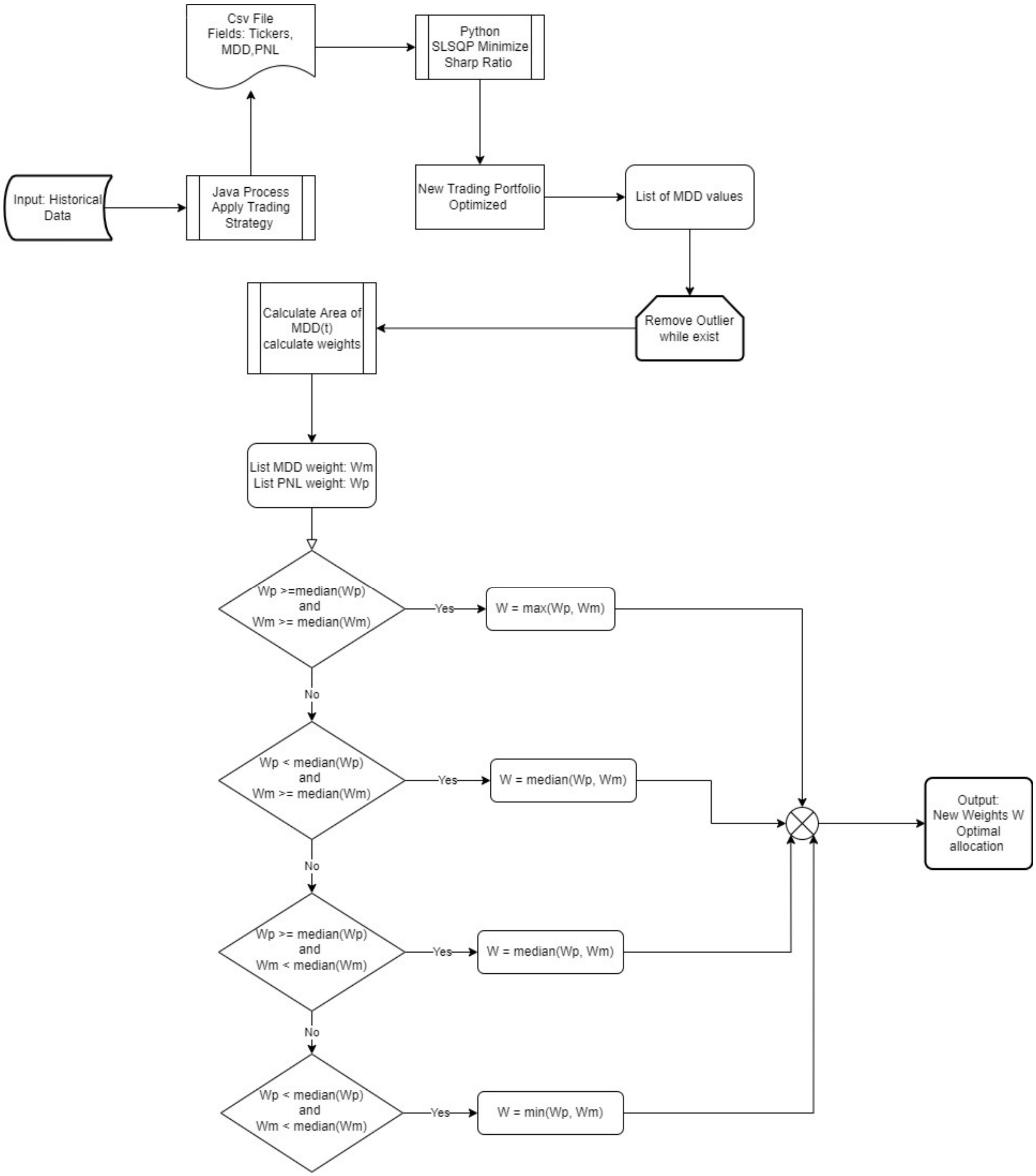


Figure 5: Flow Diagram of the optimization process

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