Hybrid Quantum Algorithms and Quantum Software Development Frameworks

Ravindra Ramouthar[1,2] and Huseyin Seker[3]

[1] *School of Digital, Technologies and Arts, Staffordshire University, Stoke-on-Trent, Staffordshire, UK*
[2] *Microsoft, Redmond, Washington, 98052, USA*
[3] *Faculty of Computing, Engineering and the Built Environment, Birmingham City University, Birmingham, UK*

Abstract

The choreography of quantum mechanical phenomena has catalysed accelerated advances in the emergence of a quantum computational substrate paradigm that yields the potential to solve some of humanities most complex problems through quantum computing speedup requires exponential computational power that exceeds that of today's most powerful super-computers, to confront the world's most intractable computational challenges in environment, agriculture, health, energy, climate, materials science, precision medicine, autonomous vehicles in smart cities, renewable energy and problems humanity has not yet even imagined. Current quantum computational substrates are limited by circuit depth, noise processes, qubit count, and fault tolerant quantum computational substrates unlikely to be available in the near term. Hybrid quantum algorithms/Variational Quantum Algorithms (VQAs), operate by using a classical optimizer to orchestrate the parameters of a quantum circuit, have become a prominent approach to address these limitations. However, several challenges manifest that include the training, speedup. accuracy, and impactful practical application of VQAs. This paper reviews novel new frontiers in current VQAs, provides an analysis of VQAs, their strengths and limitations, and outlines potential applications in the quest towards yielding practical quantum advantage. Secondly, this paper also reviews and provides an assessment of quantum software development frameworks.

1. INTRODUCTION

The near-term application of quantum computing towards short term practical applications, through reduced quantum resource requirements such as qubit count, circuit depth, numbers of gates, and numbers of measurements has become a major focus of quantum algorithm research [1]-[2]. Variational quantum algorithms (VQAs) are a promising approach to harness quantum computing for various applications that harness noisy intermediate-scale quantum (NISQ) devices. They operate by using a classical optimizer to choreograph the parameters of a quantum circuit. Researchers are predominantly proposing VQAs for all applications that are envisaged for quantum computers, and they seem to be the most likely way to achieve quantum advantage [3]. However, VQAs face some difficulties, such as how to train, optimise and speed up the quantum circuits. According to [1], algorithm hybridization remains a potent strategy towards realising near term practical applications. Algorithm hybridization is the transformation of a pure quantum algorithm into a quantum-classical algorithm which has the benefit of reduced resource requirements and increased accuracy. To overcome the intensive resource requirements posed by pure quantum algorithms that are beyond the capabilities of near-term quantum devices, a hybrid quantum-classical optimisation scheme known as the “variational quantum eigensolver” (VQE) was developed in 2014 by Peruzzo and McClean et al. [4]. The VQE algorithm is a hybrid quantum-classical algorithm that seeks variational solutions to eigenvalue and optimisation problems that are beyond reach of classical compute capability [4]-[5], is also capable of execution on any quantum device that features variational quantum error suppression whilst being able to optimise for the strengths of a given architecture through quantum hardware ansatz design for higher fidelity. [6] provides a review that seeks to unravel the rich literature and present a clear overview of the advances made on the various components of the VQE algorithm, as well as to identify key research directions that are essential for the VQE to fulfil its potential. In expanding the theory of VQE algorithms, the algorithmic improvements demonstrated by [4] include quantum variational error suppression and demonstrate the realisation of computational savings of three orders of magnitude through derivative free optimisation techniques compared with previously implemented optimisation techniques.

2. HYBRID QUANTUM ALGORITHMS

2.1 VARIATIONAL QUANTUM ALGORITHMS

Variational hybrid quantum-classical algorithms represent a class of quantum-classical algorithms with immense potential for the short-term realisation of practical applications that can be implemented on quantum computers. The implementation of variational hybrid algorithms seeks to minimise the cost function through a cost evaluation of gate sequencing on a quantum computer, which a classical computer subsequently uses as input information to make gate sequencing adjustments to parameters [1]. Shao [7] demonstrates the application of variational quantum algorithms (VQAs) towards training neural networks and solving convex optimisation problems where an exponential speedup against the number of samples and polynomial speedup against dimension of the samples over classical training algorithms is achieved. Whilst variational hybrid algorithms hold great promise for their near-term application through quantum computing, noise still limits the quality of the results. Barron and Wood [8] apply the error mitigation techniques introduced by [9] to variational hybrid algorithms called Continuous-Time Markov-Process Error Mitigation (CTMP-EM) and show that CTMP-EM is essential towards improving the performance of VQAs on near-term devices.

3. SCOPE OF REVIEW

VQAs explored in this review include the VQE, Quantum Approximate Optimisation Algorithm (QAOA), Variational Quantum Factoring (VQF), Variational Quantum Generator (VQG), Variational Quantum Compiling (VQC), Variational Quantum Linear Solver (VQLS) and Quantum Neural Networks (QNNs). *Table 1: Summary of key variational quantum algorithms*, provides a review and summary of applications, strengths and limitations of existing variational hybrid algorithms.

VQAs explored in this review include the VQE, Quantum Approximate Optimisation Algorithm (QAOA), Variational Quantum Factoring (VQF), Variational Quantum Generator (VQG), Variational Quantum Compiling (VQC), Variational Quantum Linear Solver (VQLS) and Quantum Neural Networks (QNNs). For each of these VQAs, the following key characteristics were reviewed:

1. Description
2. Application
3. References
4. Strengths
5. Limitations

Table 1: Summary of key variational quantum algorithms, provides a review and summary of applications, strengths and limitations of existing variational hybrid algorithms.

TABLE 1: SUMMARY OF KEY VARIATIONAL QUANTUM ALGORITHMS

Variational Hybrid Quantum Algorithm	**Description**	**Application**	**Reference**	**Strengths**	**Limitations**
Variational Quantum Eigensolver (VQE)	Building on the quantum simulation algorithms of [10] - Harnesses both quantum and classical resources in seeking variational solutions to eigenvalue and optimisation problems that are classically intractable.	Originally implemented on a photonic quantum chip and extended to ion trap and nitrogen vacancies in diamond quantum computers [4]. Applied in quantum chemistry for exact computations in polynomial time [11]-[12].	[11], [12], [5]	Operates on any quantum device. Variational suppression of some form of quantum errors. Optimises for given architecture.	For classical optimisation problems, noise rates and architecture-problem topology alignment remain inhibitors to quantum advantages [13].
Quantum Approximate Optimisation Algorithm (QAOA)	QAOA seeks approximate solutions to combinatorial optimisation problems [14]. Cannot be efficiently simulated on a classical device [16].	[14] investigate the application to MaxCut on regular graphs and show that the QAOA finds a cut at least 0.6924 times the size of the optimal cut. [15] apply QAOA to MaxCut problems and the Maximum Independent Set (MIS) problem, which is also NP-hard.	[14] – [16]	QAOA demonstrates a form of quantum supremacy [16]. According to [92], for certain classes of problems QAOA is able to *"outperform the best unconditionally proven classical algorithms, even when taking into account the training of the model…"*	Reachability deficits described by [17] highlight that problem density limits the algorithms capacity to minimize a corresponding objective function. Simulations of Farhi's QAOA algorithms against real-world problems modelled with realistic noise do not perform well on current NISQ systems [18].
Quantum Neural	Neural network models that harness the principles of	[19] - Quantum Associative Memory Network – Image and object	[19] - [21]	[19] presents a review of 25 significant research papers	Denormalisation of quantum states by a non-unitary operator

Networks (QNN)	quantum mechanics that seeks to develop more efficient algorithms through a combination of classical artificial neural networks with quantum information principles and generally developed as feed-forward networks [60] – [62].	recognition, pattern recognition, diagnosis of tropical diseases. Quantum Backpropagation Network – Image compression, image restoration from noise and blur. Quantum Feedforward Network – Epileptic seizure detection, recognition of handwritten numerals. Qubit-Like Neuron – Image compression, night vision processing. Two quantum machine-learning algorithms that are designed to pick out features in large data set make for a promising path to quantum advantage [22].		that describes the evolution of quantum neural network research from 1995-2020 together with a presentation on quantum neuron and network models that highlight their advantages over classical models.	which have to be normalised [19]. Linear nature of quantum neural networks in comparison to classical neural networks which are non-linear [19].
Variational Quantum Linear Solver (VQLS)	Variational hybrid quantum-classical algorithm for solving linear systems through reduced circuit depth whilst performing most of the computation classically.	Electromagnetic scattering [23]. Linear differential equation solving [24]. Finite element method [25]. Least-squares fitting [26]. Machine learning and big data analysis [27].	[28] – [30]	Estimation of cost functions which are classically intractable and problem size scaling [28].	Noise causes the training landscape to have a barren plateau [31].
Variational Quantum Factoring (VQF)	[32] present an alternative to Shor's algorithm, the VQF algorithm which maps the factoring problem to the ground state of an Ising Hamiltonian where the VQF executes through	Public key cryptography and encryption schemes [32]. Experiential work that demonstrates factoring using VQF on a superconducting quantum processor [33].	[32], [34]-[35]	VQF may be able to outperform Shor's algorithm within a decade [32].	VQF shares the strengths and limitations of QAOA [32]. VQF requires a large number of 2-qubit gates which results in a deep circuit overhead [36].

	simplifying equations over Boolean variables during pre-processing that reduces the number of qubits needed for the Hamiltonian, after which, through training variational circuits using the QAOA, seeks an approximate ground state of the resulting Ising Hamiltonian.				
Variational Quantum Generator (VQG)	VQGs comprises a quantum circuit that encodes a classical random variable into a quantum state, called the quantum encoder, and a parameterised variational circuit that is optimized to mimic a target probability distribution. The measuring of expectation values of a set of operators chosen at the beginning of the calculation form the basis of sample generation [37].	VQGs can generate representative data such as text, images, audio, etc. Generative adversarial quantum machine learning for continuous distributions [37] – [38].	[37], [39] – [40]	[37] show that VQGs could be trained using an adversarial learning approach, creating a blueprint for hybrid quantum-classical models towards solving classical machine learning problems.	The impact of noise on the estimation of expectation values and the barren plateau of the quantum neural network training landscapes [41].
Variational Quantum Compiling (QAQC)	VQC also known as quantum-assisted quantum compiling (QAQC) seeks to overcome the challenges faced when compiling quantum algorithms for near-term NISQ devices through the evaluation of an algorithms cost	[42] demonstrate a gradient-free and gradient-based approaches to cost minimisation in addition to compiling various one-qubit gates on IBM's and Rigetti's quantum computers into native gate alphabets. [42] have also simulated QAQC up to a problem size	[42], [43] – [45]	Shortened length of quantum program, application of QAQC to learning algorithms that compensate for a quantum computer's noise, benchmark the noise processes evolving on a quantum computer, and	Barren plateaus and cost function locality [46]. Abrupt transitions in the ability of quantum circuits to be trained to minimize objective functions [47].

	on a quantum computer. Overcoming the optimality challenge for quantum compilers to return a machine-level program through the least number of operations is crucial for NISQ devices where longer programs exhibit more errors in comparison to shorter programs avoiding such errors [42]. The VQC seeks to minimize the distance between the original program and the compiled program through quantifying this distance, which cannot be efficiently calculated on a classical computer [42].	of 9 that highlight the scalability of the cost function as well as the noise resilience of QAQC. Other applications of QAQC include algorithm depth compression, black-box compiling, noise mitigation, and benchmarking [42].		mitigating errors in NISQ devices [42].	

5. QUANTUM SOFTWARE FRAMEWORKS

This section provides a brief summary of quantum software development frameworks as highlighted in *Table 2: Quantum Software Development Frameworks*. There has been a mushrooming of open-source software for quantum computing projects spanning stages of the quantum toolchain that include quantum hardware interfaces, quantum compilers, implementations of quantum algorithms, coupled with all the diverse quantum computing paradigms that include quantum annealing and discrete and continuous-variable gate-model quantum computing.

[48] provides an evaluation of open-source software for quantum computing projects and an assessment of each projects characteristics together with a GitHub community evaluation and the evaluation results for static analysis of each project and its source code. Findings reveal that while there are a large number of projects, very few of these projects have attracted external developer contributions with shortcomings in software engineering featuring in many commercially backed frameworks.

TABLE 2: QUANTUM SOFTWARE DEVELOPMENT FRAMEWORKS

Quantum Software Development Framework	Description	Publisher	Reference
Strawberry Fields	A full-stack Python library to design, optimise and simulate continuous variable (CV) quantum optical circuits [49]	Xanadu	[49], [96]
PennyLane	Cross platform Python library for quantum machine learning, automatic differentiation, and optimisation of hybrid quantum-classical compute workloads and has plugins to ProjectQ and Qiskit [49].	Xanadu	[49], [97]
Cirq	Open-source framework for NISQ devices [49].	Google AI Quantum Team	[49], [98]
TensorFlow Quantum (TFQ)	Focussing on quantum data, TFQ is a quantum machine learning library for prototyping hybrid quantum-classical machine learning models which integrates logic and algorithms designed in Cirq [49].	Google	[49], [99]
Quantum Development Kit	Open-source framework for developing quantum applications with Q#, a high-level quantum-focused programming language. Q# and the QDK represent key components of Microsoft's quantum platform for learning and the development of commercial quantum applications [49] – [50].	Microsoft	[49], [50] - [51]
Qiskit	Open-source framework for working with OpenQASM and the IBM Q quantum processors that provides tools to create and manipulate quantum programs [52].	IBM	[52]
Forest	A suite comprising a quantum instruction language called Quil, an open-source Python library for Quil program development called pyQuil, a library of quantum programs called Grove, and a simulation environment called Quantum Virtual Machine [49].	Rigetti	[49] – [50]
Open Controls	Open-source Python package to develop and deploy error resilient quantum protocols [49].	Q-CTRL	[49]

Mitiq	Open-source framework for implementing error mitigation techniques on NISQ devices that is compatible with quantum programs targeting IBM Q's Qiskit, Google's Cirq, Rigetti's PyQuil [49].	Unitary Fund	[49]
Tequila	An extensible quantum information and learning architecture that operates on abstract data structures and supports backends from Qulacs, Qiskit, Cirq, and PyQuil [49].	Aspuru-guzik-Group	[49], [53]
QCircuits	Python library that offers quantum circuit simulation that is based on the quantum circuit model [49].	QCircuits	[49]
Yao	Open-source framework written in the Julia language designed for quantum algorithm design that features generic and differentiable programming of quantum circuits [49].	Yao	[49], [100]
Paddle Quantum	A quantum machine learning framework based on Baidu's flying paddle that supports the development and training of quantum neural networks that provides quantum machine learning development kits for quantum optimization, quantum chemistry and other cutting-edge quantum application tool sets [49].	Paddle Quantum	[49], [102]
Qulacs	A python/C++ library developed at Kyoto University and maintained by QunaSys that provides fast simulation of large, noisy, or parametric quantum circuits that offers better performance than Cirq, ProjectQ, pyQuil, Q#, Qiskit Terra, and QuPy due to the C/C++ backend as at Oct 2018 [49].	Qulacs	[49], [54]
staq	A full-stack quantum processing toolkit developed in C++ that offers a quantum compiler toolkit offering tools from quantum optimizers and translators to physical mappers for quantum devices with restricted connectives [55].	softwareQ	[49], [55]
Bayesforge	Linux machine image that aggregates common machine learning frameworks, such as TensorFlow, PyTorch and PyMC, with open source QC software from D-Wave, Rigetti, IBM Quantum Experience, Cirq, as well as other advanced QC frameworks [49].	Bayesforge	[49]

Blueqat	Blueqat is a Python based software framework for universal quantum computing designed for which is also designed to connect the Blueqat backend to the Nvidia CUDA based universal model simulator.	blueqat	[49], [103]
Amazon Braket	Amazon Braket is a fully managed quantum computing service that enables researchers and developers to accelerate research and discovery. Amazon Braket provides a development environment to explore and build quantum algorithms, test them on quantum circuit simulators, and run them on different quantum hardware technologies [56].	Amazon	[56]
Quantum Programming Studio	Quantum Programming Studio is a web based graphical user interface allowing users to construct quantum algorithms and obtain results by simulating directly in browser or by executing on real quantum computers.	Quantum Programming Studio	[49]
Atos/SFTC Hartree Centre – Quantum Learning as a Service (QLaaS)	Atos partnered with the UK's Science & Technology Facilities Council (SFTC) Hartree Centre offer cloud access to the Atos Quantum Learning Machine which a high performance classically based simulator that can simulate up to 38 qubits and can include quantum noise models to understand how a program would run on an actual quantum machine.	Atos	[49]
Quantum User Interface (QUI)	An initiative of the University of Melbourne offering an intuitive programming and simulation environment called QUI which is designed to enable users to visualize and understand the inner workings of a quantum computer featuring an ability to display visualizations of the quantum computer's state at every stage in the program and supports up to five qubits.	The University of Melbourne	[49]
Qibo	Qibo is an open-source high-level API provided by Qilimanjaro, written in Python and capable of executing quantum algorithms on top of different quantum computers and simulators.	Qilimanjaro	[49], [101]
XACC	XACC (eXtreme-scale ACCelerator) is a programming model and software framework enabling quantum acceleration within standard or HPC software workflows that adopts a coprocessor machine model that is independent of the	XACC	[49], [104]

	underlying quantum computing hardware which enables quantum programs to be defined and executed on a variety of QPUs types.		
Quantum++	Quantum++ is general-purpose multi-threaded quantum computing library written in C++11 and composed exclusively of header files which is not restricted to qubit systems or specific quantum information processing tasks, being capable of simulating arbitrary quantum processes.	SoftwareQ Inc.	[49]
Quantum Inspire	Provides access to technologies to perform quantum computations and learn insights into the principles of quantum computing featuring a variety of ways for users to program quantum algorithms, execute these algorithms and examine the results including a graphical interface to program in QASM (Quantum Assembly Language) and visualize operations in circuit diagrams.	QuTech	[49]
QUCAT	Open source Python library providing standard analysis tools for superconducting electronic circuits that are built around at least one Josephson junction.	QUCAT	[49]
QuTiP: Quantum Toolbox in Python	Open-source software that simulates the dynamics of open quantum systems which depends on the Numpy, Scipy and Cython numerical packages which aims to provide efficient numerical simulations of a wide variety of Hamiltonians, including those with arbitrary time-dependence.	QuTiP	[49]
OpenFermion	Open-source chemistry package for quantum computers which can be used to generate and compiling physics equations which describe chemical and material systems representations that can be interpreted by a quantum computer.	OpenFermion	[49]
Qbsolv from D-Wave	Provides tools that takes large Quadratic Unconstrained Binary Optimization (QUBO) problems and partitions them into smaller sub-QUBOs.	D-Wave	[49]
Raytheon BBN Open-Source Software	Raytheon BBN is make available three open source software programs related to Quantum Computing: Qlab (framework for superconducting qubit systems), PySimulator (A python/C++ framework for master equation simluation of qubit systems) and PyQLab (A python framework for superconducting qubit systems).	Raytheon BBN Technologies - Quantum Group	[49]
OpenQL	High-level quantum programming language and associated quantum compiler.	OpenQL	[57]

4.1 QUANTUM PROGRAMMING LANGUAGES

A survey of quantum programming languages is presented by [58]. Zhao [58] defines the term ‘quantum software engineering’ and summarises the various phases of the quantum software development life cycle. In another survey, quantum computing programming languages are classified by level of programming providing a classification of some of the existing quantum computing languages based on the level of programming [59]. In a survey paper by [98], [105]-[106], an overview is provided of state of the art in the field of quantum programming languages that focusses on high-level quantum programming languages for quantum computers, their features and comparisons. A systematic literature review (SLR) has been conducted by [107] to investigate (i) architectural process, (ii) modelling notations, (iii) architecture design patterns, (iv) tool support, and (iv) challenging factors for quantum software architecture. The results of their study suggest that quantum software represents a new category of software-intensive systems. However, existing processes and notations can be adapted to derive quantum software development architecting activities towards developing modeling languages for quantum software.

6. DISCUSSION

The classical simulation of large quantum systems or solutions to large-scale linear algebra problems remain classically intractable given their high computational overhead. A quantum computational substrate yields the potential to overcome these challenges, although fault-tolerant quantum computational substrates remain a long-term pursuit. Currently available quantum hardware bear serious limitations which include limited qubit numbers and noise processes that constrain circuit depth. VQAs which leverage a classical optimizer to train a parametrized quantum circuit, present a promising strategy to address these limitations. VQAs are now the leading approach for all applications that researchers have envisioned for quantum computers with the imminent possibility of obtaining quantum advantage [3], acknowledging that some major hurdles remain to be overcome that include the trainability, accuracy, and efficiency of VQAs. [3] present a detailed overview of the field of VQAs and strategies to address the limitations whilst highlighting the exciting prospects for harnessing VQAs as a means to obtain quantum advantage.

Other VQAs which are derivatives of those highlighted in Table 2 include:

- Variational Trace Distance Estimation Algorithm (VTDE) - employs parameterised quantum circuits (PQCs) to estimate the trace norm of an arbitrary Hermitian matrix with applications in trace distance estimation proving to be practical and efficient for NISQ devices and equivalent to VQE [63].
- Variational Fidelity Estimation Algorithm (VFE) – estimates fidelity between two quantum states [63].
- Variational Quantum State Diagonalization Algorithm (VQSD) – [1] present the VQSD algorithm for quantum state diagonalisation for applications in condensed matter physics and principal component analysis in machine learning.

5.1 APPLICATIONS

Work done by [64] towards the near-term applications of quantum computing through a quantum-classical paradigm articulates the architectural requirements of a quantum-cloud platform with a supporting framework that benchmarks its runtime performance that is optimised for variational hybrid algorithms. Key features of [64] work include parametric compilation and active qubit reset to support variational hybrid algorithms, which when integrated into the Rigetti Quantum Cloud Services (QCS) platform yields significant improvements in algorithm runtime latencies over first-generation of quantum cloud offerings. In applying the variational hybrid algorithm paradigm to machine learning, [2] present a framework where machine learning models for supervised learning and generative modelling are represented as PQCs. With generative modelling, synthetic data is generated for an unsupervised learning task through modelling unknown probability distributions and envisioned to be instrumental in the development of artificial general intelligence [2]. PQCs, made up of fixed and adjustable gates, offer a solid foundation for algorithm implementation and the demonstration of quantum supremacy in the NISQ era.

Towards demonstrating quantum advantage on applications, problems are framed as variational optimisation problems in seeking approximate solutions which have proven to be successful in combinatorial optimisation and machine learning where NISQ hardware addresses the classically intractable components of a problem [2]. The QAOA features shallow depth and is designed to run on gate-based quantum computers which receives a combinatorial optimization problem as input and outputs a string that meets a high percentage of the maximum number of satisfiable clauses [14], [16]. In analysing the performance of QAOA on MaxCut problems, [15] show an exponential improvement in the approximation ratio performance for random unweighted (or weighted) graphs.

QNNs, based on the founding ideas of [20] and [21], combine classical machine learning artificial neural network models with the principles of quantum mechanics towards more efficient algorithms [65] - [67], [69] – [74]. [19], highlighting the work of [68] which shows that, without any loss of generality, a QNN can learn faster than the control network of the same architecture. Quantum neural network models include the Quantum Associative Memory Network (QAMN), Quantum Backpropagation Neural Network (QBPN), Quantum Feedforward Neural Network (QFFNN), Quantum Competitive Neural Network (QCNN), Quantum Mcculloch Pitts Neuron (QMPN), Quantum Perceptron Neuron (QPN), Variational Quantum Circuit Neuron (VQCN), Qubit-Like Neuron (QBL) [19].

A pure quantum algorithm for solving linear systems of equations, formulated in 2009, seeks to estimate the result of a scalar measurement against a solution vector to a linear system of equations and offers exponential speedup over the fastest classical algorithms [75]. Given the limitations inherent in the near-term implementation of pure quantum algorithms towards solving systems of linear equations due to the circuit depth required [44], [44] proposes a VQLS for efficiently solving systems of linear equations on near-term devices, successfully implementing the VQLS on a problem size of 1024 x 1024 whilst [28] implemented the VQLS on a problem size of 32 × 32 using Rigetti’s quantum computer.

Peter Shor’s algorithms for the integer factorisation discrete logarithm problem are solved in polynomial time in comparison with super-polynomial time for classical algorithms [34]-[35]. However, factoring through Shor’s algorithm remains beyond the capabilities of current NISQ devices. [32] present an alternative to Shor’s algorithm, the VQF algorithm which maps the factoring problem to the ground state of an Ising Hamiltonian where the VQF executes through simplifying equations over Boolean variables during pre-processing that reduces the number of qubits needed for the Hamiltonian, after which, through training variational circuits using the QAOA, seeks an approximate ground state of the resulting Ising Hamiltonian.

VQGs comprises a quantum circuit that encodes a classical random variable into a quantum state, called the quantum encoder, and a parameterised variational circuit that is optimized to mimic a target probability distribution. The measuring of expectation values of a set of operators chosen at the beginning of the calculation form the basis of sample generation [37], [39]-[40].

VQC also known as quantum-assisted quantum compiling (QAQC) seeks to overcome the challenges faced when compiling quantum algorithms for near-term NISQ devices through the evaluation of an algorithms cost on a quantum computer. Overcoming the optimality challenge for quantum compilers to return a machine-level program through the least number of operations is crucial for NISQ devices where longer programs exhibit more errors in comparison to shorter programs avoiding such errors [42]. The VQC seeks to minimize the distance between the original program and the compiled program through quantifying this distance, which cannot be efficiently calculated on a classical computer [42].

7. CONCLUSION

There is a rapid pace of evolution taking place across the quantum computing landscape, driven by large technology vendor platforms seeking to extend their traditional offerings into the quantum computing race. According to [76], in 2023, IBM will unveil Condor, the first universal quantum computer with over a thousand qubits. The company also plans to introduce Heron, a new kind of modular quantum processor that could pave the way for quantum computers with more than 4,000 qubits by 2025. [77] maps many of the key players in the quantum computing ecosystem and [78] – [81] assesses the present quantum computing landscape. In summary, the principles of quantum mechanics and the quantum mechanical phenomena of superposition and entanglement, are permitting quantum computing to perform computations which are much more efficient than classical AI algorithms, and exponentially faster [82] – [90]. However, in the cloud of the projected hype, a key question remains. What are the impactful applications that can manifest a practical quantum advantage. An analysis by [91] reveals that many of the prospective application areas popularized will not benefit from quantum computing unless the algorithms are significantly improved [91].

Variational algorithms are a popular choice for quantum hardware in the NISQ era because of their noise tolerance. However, the vast literature on these algorithms is expansive and complex, with many different techniques and variations. Within the diversity of research, it remains difficult to find a clear and comprehensive guide on how to choreograph and optimize them. Furthermore, the practical benefits and applications of the algorithm are not well established or benchmarked for the realisation of practical quantum advantage. While some theoretical results suggest that variational algorithms can scale very well, other studies indicate that they may face significant challenges to outperform classical algorithmic methods.

REFERENCES

[1] LaRose, R., Tikku, A., O'Neel-Judy, É., Cincio, L. and Coles, P.J., 2019. Variational quantum state diagonalization. npj Quantum Information, 5(1), pp.1-10.
[2] Benedetti, M., Lloyd, E., Sack, S. and Fiorentini, M., 2019. Parameterized quantum circuits as machine learning models. Quantum Science and Technology, 4(4), p.043001.
[3] Cerezo, M., Arrasmith, A., Babbush, R., Benjamin, S.C., Endo, S., Fujii, K., McClean, J.R., Mitarai, K., Yuan, X., Cincio, L. and Coles, P.J., 2021. Variational quantum algorithms. Nature Reviews Physics, 3(9), pp.625-644.
[4] McClean, J.R., Romero, J., Babbush, R. and Aspuru-Guzik, A., 2016. The theory of variational hybrid quantum-classical algorithms. New Journal of Physics, 18(2), p.023023.
[5] Moll, N., Barkoutsos, P., Bishop, L. S., Chow, J. M., Cross, A., Egger, D. J., Filipp, S., Fuhrer, A., Gambetta, J. M., Ganzhorn, M., Kandala, A., Mezzacapo, A., Müller, P., Riess, W., Salis, G.,Smolin, J., Tavernelli, I., and Temme, K. (2018). Quantum optimization using variational algorithms on near-term quantum devices. Quantum Sci. Technol., 3:030503.
[6] Tilly, J., Chen, H., Cao, S., Picozzi, D., Setia, K., Li, Y., Grant, E., Wossnig, L., Rungger, I., Booth, G.H. and Tennyson, J., 2022. The variational quantum eigensolver: a review of methods and best practices. Physics Reports, 986, pp.1-128.
[7] Shao, C., 2019. Fast variational quantum algorithms for training neural networks and solving convex optimizations. Physical Review A, 99(4), p.042325.
[8] Barron, G.S. and Wood, C.J., 2020. Measurement Error Mitigation for Variational Quantum Algorithms. arXiv preprint arXiv:2010.08520.
[9] Bravyi, S., Sheldon, S., Kandala, A., Mckay, D.C. and Gambetta, J.M., 2020. Mitigating measurement errors in multi-qubit experiments. arXiv preprint arXiv:2006.14044.
[10] Abrams, D.S. and Lloyd, S., 1997. Simulation of many-body Fermi systems on a universal quantum computer. Physical Review Letters, 79(13), p.2586.
[11] Aspuru-Guzik, A., Dutoi, A.D., Love, P.J. and Head-Gordon, M., 2005. Simulated quantum computation of molecular energies. Science, 309(5741), pp.1704-1707.
[12] Peruzzo, A., McClean, J., Shadbolt, P., Yung, M.H., Zhou, X.Q., Love, P.J., Aspuru-Guzik, A. and O'brien, J.L., 2014. A variational eigenvalue solver on a photonic quantum processor. Nature communications, 5, p.4213.
[13] Franca, D.S. and Garcia-Patron, R., 2020. Limitations of optimization algorithms on noisy quantum devices. arXiv preprint arXiv:2009.05532.
[14] Farhi, E., Goldstone, J. and Gutmann, S., 2014. A quantum approximate optimization algorithm. arXiv preprint arXiv:1411.4028.
[15] Zhou, L., Wang, S.T., Choi, S., Pichler, H. and Lukin, M.D., 2018. Quantum approximate optimization algorithm: performance, mechanism, and implementation on near-term devices. arXiv preprint arXiv:1812.01041.
[16] Farhi, E. and Harrow, A.W., 2016. Quantum supremacy through the quantum approximate optimization algorithm. arXiv preprint arXiv:1602.07674.
[17] Akshay, V., Philathong, H., Morales, M.E. and Biamonte, J.D., 2020. Reachability deficits in quantum approximate optimization. Physical review letters, 124(9), p.090504.
[18] Guerreschi, G.G. and Matsuura, A.Y., 2019. QAOA for Max-Cut requires hundreds of qubits for quantum speed-up. Scientific reports, 9(1), p.6903.

[19] Chakraborty, S., Das, T., Sutradhar, S., Das, M. and Deb, S., 2020, June. An Analytical Review of Quantum Neural Network Models and Relevant Research. In 2020 5th International Conference on Communication and Electronics Systems (ICCES) (pp. 1395-1400). IEEE.
[20] Kak, S.C., 1995. Quantum neural computing. In Advances in imaging and electron physics (Vol. 94, pp. 259-313). Elsevier.
[21] Chrisley, R., 1995, August. Quantum learning. In New directions in cognitive science: Proceedings of the international symposium, Saariselka (pp. 4-9).
[22] Havlíček, V., Córcoles, A.D., Temme, K., Harrow, A.W., Kandala, A., Chow, J.M. and Gambetta, J.M., 2019. Supervised learning with quantum-enhanced feature spaces. Nature, 567(7747), pp.209-212.
[23] Clader, B.D., Jacobs, B.C. and Sprouse, C.R., 2013. Preconditioned quantum linear system algorithm. Physical review letters, 110(25), p.250504.
[24] Berry, D.W., 2014. High-order quantum algorithm for solving linear differential equations. Journal of Physics A: Mathematical and Theoretical, 47(10), p.105301.
[25] Montanaro, A. and Pallister, S., 2016. Quantum algorithms and the finite element method. Physical Review A, 93(3), p.032324.
[26] Wiebe, N., Braun, D. and Lloyd, S., 2012. Quantum algorithm for data fitting. Physical review letters, 109(5), p.050505.
[27] Rebentrost, P., Mohseni, M. and Lloyd, S., 2012. Quantum support vector machine for big feature and big data classifica tion. CoRR, vol. abs/1307.0471, 2014.
[28] Bravo-Prieto, C., LaRose, R., Cerezo, M., Subasi, Y., Cincio, L. and Coles, P., 2020. Variational quantum linear solver: A hybrid algorithm for linear systems. Bulletin of the American Physical Society, 65.
[29] Endo, S., Sun, J., Li, Y., Benjamin, S.C. and Yuan, X., 2020. Variational quantum simulation of general processes. Physical Review Letters, 125(1), p.010501.
[30] Chen, C.C., Shiau, S.Y., Wu, M.F. and Wu, Y.R., 2019. Hybrid classical-quantum linear solver using Noisy Intermediate-Scale Quantum machines. Scientific reports, 9(1), pp.1-12.
[31] Wang, S., Fontana, E., Cerezo, M., Sharma, K., Sone, A., Cincio, L. and Coles, P.J., 2020. Noise-induced barren plateaus in variational quantum algorithms. arXiv preprint arXiv:2007.14384.
[32] Anschuetz, E., Olson, J., Aspuru-Guzik, A. and Cao, Y., 2019, March. Variational quantum factoring. In International Workshop on Quantum Technology and Optimization Problems (pp. 74-85). Springer, Cham.
[33] Karamlou, A.H., Simon, W.A., Katabarwa, A., Scholten, T.L., Peropadre, B. and Cao, Y., 2020. Analyzing the Performance of Variational Quantum Factoring on a Superconducting Quantum Processor. arXiv preprint arXiv:2012.07825.
[34] Shor, P.W., 1994, May. Polynomial time algorithms for discrete logarithms and factoring on a quantum computer. In International Algorithmic Number Theory Symposium (pp. 289-289). Springer, Berlin, Heidelberg.
[35] Shor, P.W., 1999. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM review, 41(2), pp.303-332.
[36] Qiu, L., Alam, M., Ash-Saki, A. and Ghosh, S., 2020. Resiliency Analysis and Improvement of Variational Quantum Factoring in Superconducting Qubit. arXiv preprint arXiv:2004.12447.
[37] Romero, J. and Aspuru-Guzik, A., 2019. Variational quantum generators: Generative adversarial quantum machine learning for continuous distributions. arXiv preprint arXiv:1901.00848.
[38] Anand, A., Romero, J., Degroote, M. and Aspuru-Guzik, A., 2020. Experimental demonstration of a quantum generative adversarial network for continuous distributions. arXiv preprint arXiv:2006.01976.
[39] Schuld, M., Sweke, R. and Meyer, J.J., 2020. The effect of data encoding on the expressive power of variational quantum machine learning models. arXiv preprint arXiv:2008.08605.

[40] Zoufal, C., Lucchi, A. and Woerner, S., 2019. Quantum generative adversarial networks for learning and loading random distributions. npj Quantum Information, 5(1), pp.1-9.

[41] Romero, J. and Aspuru-Guzik, A., 2021. Variational quantum generators: Generative adversarial quantum machine learning for continuous distributions. Advanced Quantum Technologies, 4(1), p.2000003.

[42] Khatri, S., LaRose, R., Poremba, A., Cincio, L., Sornborger, A.T. and Coles, P.J., 2019. Quantum-assisted quantum compiling. Quantum, 3, p.140.

[43] Sharma, K., Khatri, S., Cerezo, M. and Coles, P.J., 2020. Noise resilience of variational quantum compiling. New Journal of Physics, 22(4), p.043006.

[44] Kübler, J.M., Arrasmith, A., Cincio, L. and Coles, P.J., 2020. An adaptive optimizer for measurement-frugal variational algorithms. Quantum, 4, p.263.

[45] Yuan, X., Endo, S., Zhao, Q., Li, Y. and Benjamin, S.C., 2019. Theory of variational quantum simulation. Quantum, 3, p.191.

[46] Uvarov, A. and Biamonte, J., 2021. On barren plateaus and cost function locality in variational quantum algorithms. Journal of Physics A: Mathematical and Theoretical.

[47] Campos, E., Nasrallah, A. and Biamonte, J., 2021. Abrupt transitions in variational quantum circuit training. Physical Review A, 103(3), p.032607.

[48] Fingerhuth, M., Babej, T. and Wittek, P., 2018. Open source software in quantum computing. PloS one, 13(12), p.e0208561

[49] Quantum Computing Report. 2021. Tools of Quantum Computing. [ONLINE] Available at: https://quantumcomputingreport.com/tools/. [Accessed 7 January 2021].

[50] Microsoft. 2019. Azure Quantum. [ONLINE] Available at: https://azure.microsoft.com/en-gb/services/quantum/ [Accessed 06 Jan. 2020].

[51] LaRose, R., 2019. Overview and comparison of gate level quantum software platforms. Quantum, 3, p.130.

[52] Qiskit. 2021. Open-Source Quantum Development. [ONLINE] Available at: https://qiskit.org. [Accessed 7 January 2021].

[54] Suzuki, Y., Kawase, Y., Masumura, Y., Hiraga, Y., Nakadai, M., Chen, J., Nakanishi, K.M., Mitarai, K., Imai, R., Tamiya, S. and Yamamoto, T., 2020. Qulacs: a fast and versatile quantum circuit simulator for research purpose. arXiv preprint arXiv:2011.13524.

[55] Amy, M. and Gheorghiu, V., 2020. staq-A full-stack quantum processing toolkit. Quantum Science and Technology.

[56] Amazon Web Services, Inc., 2019. Amazon Braket [ONLINE] Available at: https://aws.amazon.com/braket/ [Accessed 06 Jan. 2020].

[57] Khammassi, N., Ashraf, I., Someren, J.V., Nane, R., Krol, A.M., Rol, M.A., Lao, L., Bertels, K. and Almudever, C.G., 2020. OpenQL: A portable quantum programming framework for quantum accelerators. arXiv preprint arXiv:2005.13283.

[58] Zhao, J., 2020. Quantum Software Engineering: Landscapes and Horizons. arXiv preprint arXiv:2007.07047.

[59] Quantum World Association. 2021. Quantum Computing languages landscape. [ONLINE] Available at: https://medium.com/@quantum_wa/quantum-computing-languages-landscape-1bc6dedb2a35. [Accessed 2 May 2021].

[60] Abbas, A., Sutter, D., Zoufal, C., Lucchi, A., Figalli, A. and Woerner, S., 2020. The power of quantum neural networks. arXiv preprint arXiv:2011.00027.

[61] Liao, Y., Hsieh, M.H. and Ferrie, C., 2021. Quantum Optimization for Training Quantum Neural Networks. arXiv preprint arXiv:2103.17047.

[62] Shen, H., Zhang, P., You, Y.Z. and Zhai, H., 2020. Information scrambling in quantum neural networks. Physical Review Letters, 124(20), p.200504.

[63] Chen, R., Song, Z., Zhao, X. and Wang, X., 2020. Variational Quantum Algorithms for Trace Distance and Fidelity Estimation. arXiv preprint arXiv:2012.05768.

[64] Karalekas, P.J., Tezak, N.A., Peterson, E.C., Ryan, C.A., da Silva, M.P. and Smith, R.S., 2020. A quantum-classical cloud platform optimized for variational hybrid algorithms. Quantum Science and Technology, 5(2), p.024003.

[65] da Silva, A.J., Ludermir, T.B. and de Oliveira, W.R., 2016. Quantum perceptron over a field and neural network architecture selection in a quantum computer. Neural Networks, 76, pp.55-64.

[66] Schuld, M., Sinayskiy, I. and Petruccione, F., 2014. The quest for a quantum neural network. Quantum Information Processing, 13(11), pp.2567-2586.

[66] Panella, M. and Martinelli, G., 2011. Neural networks with quantum architecture and quantum learning. International Journal of Circuit Theory and Applications, 39(1), pp.61-77.

[67] Schuld, M., Sinayskiy, I. and Petruccione, F., 2015. An introduction to quantum machine learning. Contemporary Physics, 56(2), pp.172-185.

[68] Menneer, T. and Narayanan, A., 1995. Quantum-inspired neural networks. Tech. Rep. R329.

[69] Gupta, S. and Zia, R.K.P., 2001. Quantum neural networks. Journal of Computer and System Sciences, 63(3), pp.355-383.

[70] Ezhov, Alexandr A., and Ventura, D., 2000. Quantum Neural Networks. In Future Directions for Intelligent Systems and Information Sciences: The Future of Speech and Image Technologies, Brain Computers, pp213–235.

[71] Beer, K., Bondarenko, D., Farrelly, T., Osborne, T.J., Salzmann, R., Scheiermann, D. and Wolf, R., 2020. Training deep quantum neural networks. Nature communications, 11(1), pp.1-6.

[72] Beer, K., Bondarenko, D., Farrelly, T., Osborne, T.J., Salzmann, R. and Wolf, R., 2019. Efficient learning for deep quantum neural networks. arXiv preprint arXiv:1902.10445.

[73] Beer, K., List, D., Müller, G., Osborne, T.J. and Struckmann, C., 2021. Training Quantum Neural Networks on NISQ Devices. arXiv preprint arXiv:2104.06081.

[74] Skolik, A., McClean, J.R., Mohseni, M., van der Smagt, P. and Leib, M., 2021. Layerwise learning for quantum neural networks. Quantum Machine Intelligence, 3(1), pp.1-11.

[75] Harrow, A.W., Hassidim, A. and Lloyd, S., 2009. Quantum algorithm for linear systems of equations. Physical review letters, 103(15), p.150502.

[76] Choi, C.Q., 2023. IBM's Quantum Leap: The Company Will Take Quantum Tech Past the 1,000-Qubit Mark in 2023. IEEE Spectrum, 60(1), pp.46-47.

[77] Silicon Foundry. 2021. Near-Future: Quantum Computing. [ONLINE] Available at: https://sifoundry.medium.com/near-future-quantum-computing-caae89b0498e. [Accessed 2 May 2021].

[78] MacQuarrie, E.R., Simon, C., Simmons, S. and Maine, E., 2020. The emerging commercial landscape of quantum computing. Nature Reviews Physics, 2(11), pp.596-598.

[79] Hassija, V., Chamola, V., Saxena, V., Chanana, V., Parashari, P., Mumtaz, S. and Guizani, M., 2020. Present landscape of quantum computing. IET Quantum Communication, 1(2), pp.42-48

[80] Deshpande, A., 2022. Assessing the quantum-computing landscape. Communications of the ACM, 65(10), pp.57-65.

[81] Satyanarayanan, M., Gao, W. and Lucia, B., 2019, February. The computing landscape of the 21st century. In Proceedings of the 20th International Workshop on Mobile Computing Systems and Applications (pp. 45-50).

[82] Sgarbas, K.N., 2007. The road to quantum artificial intelligence. arXiv preprint arXiv:0705.3360.

[83] Sriarunothai, T., Wölk, S., Giri, G.S., Friis, N., Dunjko, V., Briegel, H.J. and Wunderlich, C., 2018. Speeding-up the decision making of a learning agent using an ion trap quantum processor. Quantum Science and Technology, 4(1), p.015014.

[84] Dong, D., Chen, C., Li, H. and Tarn, T.J., 2008. Quantum reinforcement learning. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 38(5), pp.1207-1220.
[85] Paparo, G.D. and Martin-Delgado, M.A., 2012. Google in a quantum network. Scientific reports, 2(1), pp.1-12.
[86] Paparo, G.D., Dunjko, V., Makmal, A., Martin-Delgado, M.A. and Briegel, H.J., 2014. Quantum speedup for active learning agents. Physical Review X, 4(3), p.031002.
[87] Paparo, G.D., Dunjko, V., Makmal, A., Martin-Delgado, M.A. and Briegel, H.J., 2014. Quantum speedup for active learning agents. Physical Review X, 4(3), p.031002.
[88] Crawford, D., Levit, A., Ghadermarzy, N., Oberoi, J.S. and Ronagh, P., 2016. Reinforcement learning using quantum Boltzmann machines. arXiv preprint arXiv:1612.05695.
[89] Briegel, H.J. and De las Cuevas, G., 2012. Projective simulation for artificial intelligence. Scientific reports, 2(1), pp.1-16.
[90] Jerbi, S., Trenkwalder, L.M., Nautrup, H.P., Briegel, H.J. and Dunjko, V., 2021. Quantum enhancements for deep reinforcement learning in large spaces. PRX Quantum, 2(1), p.010328.
[91] Hoefler, T., Häner, T. and Troyer, M., 2023. Disentangling hype from practicality: on realistically achieving quantum advantage. Communications of the ACM, 66(5), pp.82-87.
[92] Anschuetz, E.R. and Kiani, B.T., 2022. Quantum variational algorithms are swamped with traps. Nature Communications, 13(1), p.7760.
[93] Zhang, S. and Li, L., 2022. A brief introduction to quantum algorithms. CCF Transactions on High Performance Computing, 4(1), pp.53-62.
[94] Du, Y., Tu, Z., Yuan, X. and Tao, D., 2022. Efficient measure for the expressivity of variational quantum algorithms. Physical Review Letters, 128(8), p.080506.
[95] Weigold, M., Barzen, J., Leymann, F. and Vietz, D., 2021. Patterns for hybrid quantum algorithms. In Service-Oriented Computing: 15th Symposium and Summer School, SummerSOC 2021, Virtual Event, September 13–17, 2021, Proceedings 15 (pp. 34-51). Springer International Publishing
[96] Killoran, N., Izaac, J., Quesada, N., Bergholm, V., Amy, M. and Weedbrook, C., 2019. Strawberry fields: A software platform for photonic quantum computing. Quantum, 3, p.129.
[97] Bergholm, V., Izaac, J., Schuld, M., Gogolin, C., Ahmed, S., Ajith, V., Alam, M.S., Alonso-Linaje, G., AkashNarayanan, B., Asadi, A. and Arrazola, J.M., 2018. Pennylane: Automatic differentiation of hybrid quantum-classical computations. arXiv preprint arXiv:1811.04968.
[98] Heim, B., Soeken, M., Marshall, S., Granade, C., Roetteler, M., Geller, A., Troyer, M. and Svore, K., 2020. Quantum programming languages. Nature Reviews Physics, 2(12), pp.709-722.
[99] Broughton, M., Verdon, G., McCourt, T., Martinez, A.J., Yoo, J.H., Isakov, S.V., Massey, P., Halavati, R., Niu, M.Y., Zlokapa, A. and Peters, E., 2020. TensorFlow Quantum: A Software Framework for Quantum Machine Learning.
[100] Luo, X.Z., Liu, J.G., Zhang, P. and Wang, L., 2020. Yao. jl: Extensible, efficient framework for quantum algorithm design. Quantum, 4, p.341.
[101] Efthymiou, S., Ramos-Calderer, S., Bravo-Prieto, C., Pérez-Salinas, A., García-Martín, D., Garcia-Saez, A., Latorre, J.I. and Carrazza, S., 2021. Qibo: a framework for quantum simulation with hardware acceleration. Quantum Science and Technology, 7(1), p.015018.
[102] Singh, M., Dhara, C., Kumar, A., Gill, S.S. and Uhlig, S., 2022. Quantum artificial intelligence for the science of climate change. In Artificial Intelligence, Machine Learning and Blockchain in Quantum Satellite, Drone and Network (pp. 199-207). CRC Press.
[103] Ishiyama, Y., Nagai, R., Mieda, S., Takei, Y., Minato, Y. and Natsume, Y., 2022. Noise-robust optimization of quantum machine learning models for polymer properties using a simulator and validated on the IonQ quantum computer. Scientific Reports, 12(1), p.19003.

[104] McCaskey, A.J., Lyakh, D.I., Dumitrescu, E.F., Powers, S.S. and Humble, T.S., 2020. XACC: a system-level software infrastructure for heterogeneous quantum–classical computing. Quantum Science and Technology, 5(2), p.024002.
[105] Garhwal, S., Ghorani, M. and Ahmad, A., 2021. Quantum programming language: A systematic review of research topic and top cited languages. Archives of Computational Methods in Engineering, 28, pp.289-310.
[106] Selinger, P., 2004, April. A brief survey of quantum programming languages. In International Symposium on Functional and Logic Programming (pp. 1-6). Berlin, Heidelberg: Springer Berlin Heidelberg.
[107] Khan, A.A., Ahmad, A., Waseem, M., Liang, P., Fahmideh, M., Mikkonen, T. and Abrahamsson, P., 2023. Software architecture for quantum computing systems—A systematic review. Journal of Systems and Software, 201, p.111682.