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How do discrete optimization models (MST, Steiner Tree, Max-Flow/Min-Cut, MILP) scale in cost, coverage, and runtime for rural electrification networks of different sizes?

I. INTRODUCTION

A. Resource Allocation in Rural Electrification

Rural electrification involves distributing limited resources such as transmission lines, transformers, or solar mini-grids across dispersed communities. Unlike urban areas, where demand is concentrated and infrastructure already exists, rural settlements are scattered, making allocation decisions both technical and strategic. Efficient allocation maximizes coverage, minimizes cost, and ensures equitable access, while inefficient planning can leave some villages underserved and waste resources [1].

The challenges of rural electrification resemble those encountered in networked resource allocation problems in supply chains and logistics, where connectivity, capacity constraints, and cost must all be considered simultaneously [2][3]. In this context, mathematical optimization provides a structured framework for decision-making, allowing planners to evaluate alternative configurations, anticipate bottlenecks, and quantify trade-offs.

B. Optimization Approaches for Network Design

Several discrete optimization techniques have been applied to rural electrification:

1. Minimum Spanning Tree (MST) methods connect all nodes with the minimum total edge cost, ensuring network connectivity with minimal infrastructure investment [1].

2. Steiner Tree models extend MST by introducing additional nodes, potentially reducing total cost at the expense of coverage [8].

3. Max-Flow/Min-Cut (MFMC) frameworks evaluate the maximum electricity flow through a network and identify minimal cuts that could disconnect the system, highlighting capacity constraints and potential bottlenecks [10].

4. Mixed Integer Linear Programming (MILP) integrates discrete construction decisions and continuous flows, optimizing both network design and operational constraints under budget or technical limits [7].

These models provide complementary perspectives: MST and Steiner Tree prioritize cost efficiency, MFMC emphasizes functional capacity, and MILP allows multi-objective optimization under constraints.

C. Scalability and Research Gap

While these models have been applied successfully to small-scale networks (typically fewer than 100 nodes), their performance in larger regional or national networks remains less explored. Increased node counts, longer distances, and variable demand pose challenges for runtime, memory usage, and coverage. Erdogdu [9] emphasizes that careful formulation of optimization algorithms is critical for efficiently solving large-scale network problems, which is directly relevant for rural electrification planning.

Existing studies often examine individual models in isolation [1][7][8][10], but systematic comparisons of MST, Steiner Tree, MFMC, and MILP across small and large scales are limited. This creates a need for research that evaluates trade-offs between computational efficiency, cost, and coverage as system size increases.

D. Aim and Research Questions

This study evaluates the scalability of four discrete optimization models—MST, Steiner Tree, MFMC, and MILP—across networks of 50 and 1000 nodes. The goal is to identify how each model behaves under increasing scale and to provide insights into model selection for rural electrification planning. Specifically, the research addresses:

1. How do graph-based optimization models scale in runtime and cost as network size increases?

2. To what extent can MILP and network flow models optimize infrastructure placement while maintaining coverage?

3. What trade-offs emerge between minimizing investment, maximizing service coverage, and ensuring computational feasibility at different scales?

By investigating these questions, the study provides practical guidance for planners and demonstrates how classical discrete optimization can be applied to large-scale rural electrification networks, balancing efficiency, reliability, and equity.

II. THEORETICAL BACKGROUND

A. Discrete Mathematics and Infrastructure Decisions

Rural electrification planning involves discrete allocation decisions, such as whether to construct a distribution line, place a transformer, or install a solar mini-grid at a particular location. These decisions are inherently binary or integer, making discrete mathematics and combinatorial optimization highly relevant.

Key mathematical tools used in this context include:

1. Graph Theory: Villages or households are modeled as nodes, and possible distribution connections are modeled as edges. Graph representations allow systematic evaluation of connectivity, flow capacity, and network cost [1][8].

2. Combinatorial Methods: These evaluate alternative layouts and resource allocations, identifying solutions that minimize cost while maximizing coverage [2][3].

3. Integer and Mixed-Integer Programming (MILP): MILP frameworks integrate discrete infrastructure decisions with continuous electricity flows, enabling optimization under budget and capacity constraints [7].

These methods have been successfully applied in logistics, telecommunications, and supply chain network design, demonstrating their capacity to manage complex resource allocation problems [1][7].

B. Graph-Theoretic Models

1. Minimum Spanning Tree (MST):
   MST algorithms determine the least-cost connections that ensure all nodes are connected to a central source. In rural electrification, MST minimizes the total length of lines

required to connect all villages, reducing initial infrastructure costs [1]. MST provides a baseline network design, prioritizing efficiency over additional redundancy or coverage optimization.

2. Steiner Tree:
   Steiner Tree models extend MST by allowing additional intermediate nodes to be introduced, which can reduce overall network cost. While this can improve efficiency, coverage may decrease if some villages are not directly connected, highlighting a trade-off between cost and reach [8].

C. Max-Flow / Min-Cut (MFMC) Models

MFMC frameworks model electricity flow through a network, allowing planners to quantify:

- The maximum power that can be delivered to all villages simultaneously.

- Critical network segments (min-cuts) where failure would isolate part of the network [10].

MFMC models are particularly useful for evaluating bottlenecks, capacity limitations, and robustness of network designs, complementing cost-focused MST and Steiner Tree models.

D. Mixed-Integer Linear Programming (MILP)

MILP integrates discrete infrastructure decisions (e.g., whether to build a line) with continuous operational flows (e.g., electricity delivered). This allows optimization of multiple objectives, such as:

- Minimizing total installation and operational cost.

- Maximizing coverage across villages.
- Maintaining feasible flow levels under capacity constraints [7].

MILP is flexible and can accommodate complex constraints, making it suitable for large-scale network planning. However, its computational requirements increase rapidly with network size, making scalability an important consideration [9].

These four discrete optimization frameworks provide complementary perspectives for rural electrification planning, allowing a comprehensive evaluation of cost, coverage, and scalability. This study investigates how each model behaves at small (50-node) and large (1000-node) scales to inform model selection for practical deployment.

III. LITERATURE REVIEW

A. Optimization in Rural Electrification

The application of discrete optimization methods in rural electrification has been extensively explored to minimize network costs and maximize service coverage. Minimum Spanning Tree (MST) approaches identify the least-cost set of edges connecting all nodes, ensuring full network connectivity with minimal infrastructure investment [1]. Steiner Tree models extend this approach by introducing additional "Steiner nodes," which allow for a reduction in total network cost at the potential expense of complete coverage [8]. These methods highlight the trade-offs between cost efficiency and network reach.

Mixed Integer Linear Programming (MILP) has been applied to off-grid microgrids and hybrid energy systems, enabling planners to integrate both discrete construction decisions (e.g., whether to build a transformer or a solar installation) and continuous variables such as energy flow [7]. MILP provides a flexible and formal framework for optimizing multiple competing objectives, including cost minimization, network coverage, and adherence to technical constraints.

Max-Flow/Min-Cut (MFMC) models have been used to evaluate network capacity and identify potential bottlenecks in electricity distribution [10]. By computing the maximum flow through the network and the minimal set of edges whose removal would disconnect the system, MFMC informs both redundancy planning and resource allocation. This approach complements MST and Steiner Tree methods by emphasizing the functional performance of a network under capacity constraints rather than purely its cost.

B. Scalability Considerations

While MST, Steiner Tree, MILP, and MFMC are well-established for small networks, their performance at larger scales is less explored. Scalability challenges include increased computational time, memory usage, and potential coverage trade-offs as the number of nodes grows. Erdogdu [9] discusses optimization algorithm design for efficiently solving large-scale integer and network problems, highlighting that solver selection and model formulation significantly impact runtime and feasibility.

In practice, small village networks (50 nodes) are generally handled efficiently by all four models, but larger regional systems (1000 nodes or more) can expose computational bottlenecks and coverage limitations. Understanding how each model scales is critical for planning, especially when resources must be allocated under strict cost and time constraints.

C. Synthesis and Gap Analysis

Previous studies [1][7][8][10] have demonstrated the utility of discrete optimization for rural electrification, yet most focus on isolated small-scale networks. Few studies systematically compare MST, Steiner Tree, MFMC, and MILP on both small and large scales to evaluate trade-offs in runtime, cost, coverage, and memory usage.

This study addresses that gap by simulating these four models across two scales—50 nodes (small network) and 1000 nodes (large network)—focusing on their scalability, efficiency, and performance trade-offs. By evaluating runtime, cost, and coverage for each method, the study provides practical guidance for planners seeking efficient and reliable resource allocation strategies.

IV. METHODOLOGY

This study employs a comparative, simulation based study to evaluate the scalability of discrete optimization techniques in rural electrification planning.

Four mathematical models are implemented, Minimum Spanning Tree (MST), Steiner Tree, Max-Flow/MIn-Cut and Mixed Integer Linear programming (MILP) - both tested at two scales:

- a small network of 50 villages, and
- a large network of 1,000 villages.

The goal is to determine how these models perform as network size increases in terms of computational efficiency, total cost and coverage (where applicable).

Optimization Models

Each model corresponds to a distinct optimization approach drawn from prior studies.

Minimum Spanning Tree (MST) Model

The MST model connects all villages with the lowest total cost while avoiding loops. It represents the simplest structure for building an electrical distribution network by ensuring every village is connected at least once, with minimum total wiring cost.

$$\min \sum_{(i,j)\in E} w_{ij} x_{ij}$$

Subject to:

- The network must be connected (every node can be reached).
- The network must be acyclic.

- Xij=1 if a line between village *i* and *j* is built, otherwise Xij=0

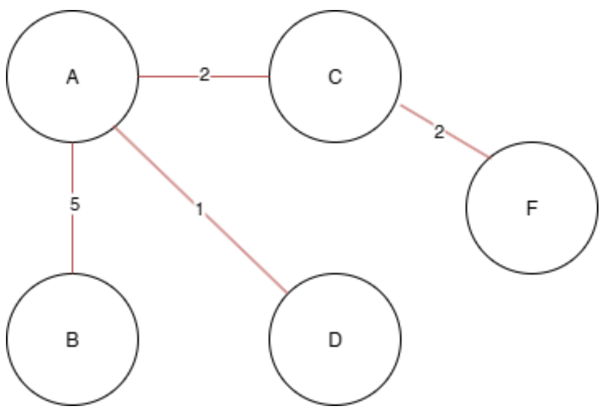
Where:

- wij: Cost of connecting village *i* and *j*
- xij: Binary decision variable

Algorithm used: Prim's and Kruskals (Rothkopf, Fichtner, & Schultmann, 2018).

Diagram 1: MST Structure

(*A simple network diagram highlighting MST edges in red (least-cost connections).*



Steiner Tree Model

The Steiner Tree model extends MST by allowing extra connection points (Steiner nodes) that don't represent real villages but reduce total network cost, like adding junction poles to make lines shorter overall. For Steiner, we are going to let V be the set of village nodes, and S the set of additional connection points (Steiner nodes).

The goal is to connect all V using some or all of S to minimize cost.

$$\min \sum_{(i,j)\in E'} w_{ij}x_{ij}$$

Subject to:

- V = set of village nodes
- S = set of additional “Steiner” nodes
- The network connects all villages using some or all Steiner nodes.

Where:

- wij: Cost of connecting nodes *i* and *j*
- xij: Binary variable

Reference: (Rothkopf et al., 2018).

Diagram 3: Steiner Tree Network

(*Visual showing extra nodes (Steiner points) that reduce total wiring cost.)*

<u>Max-Flow / Min-Cut Model</u>

This model analyzes how much power can flow through the network from a source (like a power plant) to multiple destinations (villages), and identifies bottlenecks and also points where capacity is fully used.

$$\max \sum_{(s,i)\in E} f_{si}$$

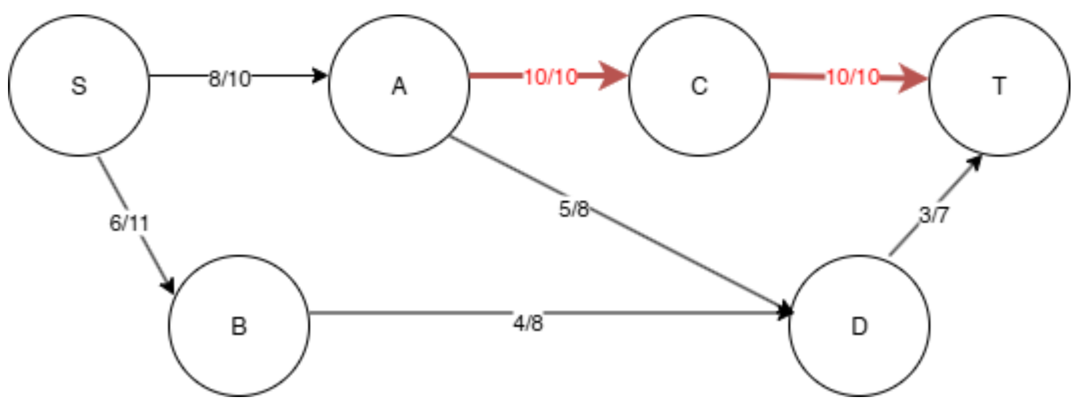
Subject to:

- $F_{ij} \leq C_{ij}$ (flow cannot exceed line capacity)
- $\sum_j f_{ij} - \sum_j f_{ji} = 0$ (flow conservation at each node, except source and sink)

Where:

- F_{ij}: Electricity flow from node i to j.
- C_{ij}: Maximum capacity of the line between i and j
- s: Source node (power station)
- t: Sink node (load or demand point)

Diagram 2: Flow Network



Directed graph showing power flow (in units) and capacity limits. Red edges indicate bottlenecks where flow equals capacity.

Mixed Integer Linear Programming (MILP) Model

MILP combines both discrete decisions (whether to build or install facilities) and continuous variables (power flow). It is the most flexible and computationally intensive model capable of balancing cost, capacity, and coverage together. ((Bischi et al., 2014).

$$\min \sum_{(i,j)} w_{ij} x_{ij} + \sum_{i} F_i y_i$$

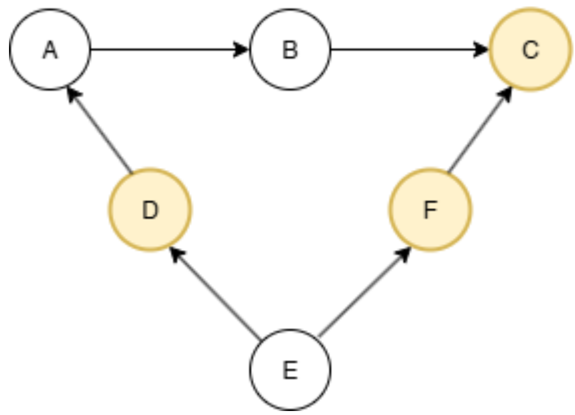
Subject to:

- $F_{ij} \leq M x_{ij}$ (flow only if line is built)
- $\sum_j f_{ij} - \sum_j f_{ji} = d_i$
- X_{ij} , $y_i \in \{0,1\}$

Where:

- w_{ij}: Line cost between i and j
- F_i: Fixed cost of installing a facility (e.g., substation or mini-grid)
- x_{ij}: 1 if line built between i and j
- y_i: 1 if facility installed at i
- f_{ij}: Power flow from i to j
- M: Large constant ensuring flow is limited by whether a line exists
- d_i: Net demand at node i

Diagram 3: MILP Resource Allocation Network



Nodes represent villages. Highlighted nodes indicate optimal facility sites selected by the MILP model. Arrows represent optimized power flows between facilities and demand nodes.

3.3 Simulation and Performance Evaluation

This study evaluates the scalability of four discrete optimization models; MST, Steiner Tree, Max-Flow/Min-Cut, and MILP by testing each individually on two network scales:

- Small Network (50 villages): used for baseline validation and accuracy verification.
- Large Network (1,000 villages): used for scalability testing in terms of computational feasibility and cost growth.

For each scale, 10 random spatial layouts of villages are generated using different seeds. Each model is executed on all instances, and the mean ± standard deviation of the following metrics are recorded: runtime (s), peak memory (MB), total connection cost (GHS), percentage of villages served, and solution quality (e.g., optimality gap for MILP).

Scale	Villages (nodes)	Purpose
Small network	50	Baseline validation (accuracy & structure)
Large network	1,000	Scalability testing (computational feasibility & cost scaling

For each model, performance is visualized through network graphs at both scales. Growth patterns in cost and runtime are analyzed to determine each model's scalability behavior.

Tools and Computational Environment

All simulations will be implemented in Python.

- NetworkX will be used for MST, Steiner, and Max-Flow/Min-Cut implementations.
- Pyomo or PuLP will be used for MILP formulation and solving.
- Solvers: CBC (open-source) or Gurobi (if available).
- Hardware: Apple MacBook Pro (M4 Pro, 24 GB RAM, 11-core CPU).
- Software Environment: macOS Sequoia, Python 3.12.

- Execution: All experiments will be run locally using JupyterLab or Google Colab for reproducibility.

For MILP experiments, graph sparsification (k-nearest neighbors, $k = 8$) will be applied to limit edge density, and each solver run will have a 2-hour time limit with a 2 % optimality gap. Solver logs, objective values, and memory usage will be recorded for every run.

5. Evaluation Metrics

Criterion	How it is Measured	Purpose
Computational Efficiency	Runtime (s), CPU time (s), peak RAM (MB)	Measures scalability with increasing network size
Solution Quality / Cost	Total connection cost (GHS), solver status, MIP gap (%)	Evaluates economic efficiency and solver performance
Coverage & Reliability	% villages served, unmet demand (kWh/day)	Assesses practical reach of each model
Scalability Ratio	Rt=t1000/t50, Rc=Z1000/Z50	Quantifies performance growth between scales
Robustness	Variation across 10 random runs; sensitivity to α (cost/	Checks stability of model outcomes

6. Validation

Before scalability testing, small-scale results (50 villages) will be cross-validated against reference values from Mandelli et al. (2016) and Akbaş et al. (2023) to confirm consistency in cost and network structure. Once validated, large-scale simulations will proceed under identical parameter settings to ensure comparability.

Regression and trend analyses will be applied to the recorded metrics to model scalability patterns. Reproducibility will be ensured by publishing code, parameters, and synthetic datasets in a public repository, along with seed values and solver logs.

V. IMPLEMENTATION FRAMEWORK

The implementation phase operationalizes the four optimization models discussed earlier, the Minimum Spanning Tree (MST), Steiner Tree, Max-Flow/Min-Cut, and Mixed Integer Linear Programming (MILP) to evaluate their scalability under increasing network sizes. Each model is coded, executed, and analyzed independently to maintain clarity of objectives, as each optimization technique targets a distinct planning challenge within rural electrification.

5.1 Minimum Spanning Tree (MST) Implementation

Objective: Connect all villages at the minimum total connection cost without forming cycles.

Algorithm: Kruskal's or Prim's algorithm (via NetworkX).

Procedure:

1. Input village coordinates and compute pairwise distances wij.
2. Build a complete weighted graph G(V,E).
3. Use the MST algorithm to identify the subset of edges that connect all nodes with minimum total weight.
4. Output: total cost, coverage, runtime, memory usage and network visualization.

Scalability test: MST is executed for both 50 and 1,000 nodes. Runtime and memory growth are recorded.

5.2 Steiner Tree Model Implementation

Objective: Further reduce total wiring cost by introducing auxiliary nodes (Steiner points) where optimal.

Algorithm: Steiner approximation via NetworkX or GeoSteiner heuristic.

Procedure:

1. Start from MST solution as baseline.
2. Identify potential intermediate points (centroids between clusters of villages).
3. Reconnect graph through these points to minimize total network length.
4. Output: new total cost, added Steiner nodes, runtime, memory usage and network visualisation.

Scalability test: Compare cost savings and computation time for 50 vs. 1,000-node networks.

5.3 Max-Flow / Min-Cut Model Implementation

Objective: Evaluate how much electrical power can flow through the network and identify bottlenecks (capacity limits).

Algorithm: Ford–Fulkerson algorithm for maximum flow; corresponding min-cut identification.

Procedure:

1. Define one source (main grid or generation hub) and several sink nodes (villages).
2. Assign edge capacities proportional to line size or reliability rating.
3. Compute the maximum possible flow from source to all sinks.
4. Identify edges forming the minimum cut (bottlenecks).
5. Output: max flow value, cut-set edges, and capacity utilization ratio.

Scalability test: Observe how total flow and bottleneck ratio change from 50 to 1,000 nodes.

5.4 Mixed Integer Linear Programming (MILP) Implementation

Objective: Jointly decide which facilities to build and how power should flow through the network to minimize cost.

Formulation: $\text{Minimize } Z = \sum_{i,j} c_{ij} x_{ij} + \sum_{i} F_i y_i$

Subject to:

- Power balance at each node
- Capacity constraints on edges
- Binary investment decisions xij, yi∈{0,1}
- Demand satisfaction fij ≥ di

Procedure:

1. Define binary variables for line and facility installation.
2. Define continuous variables for power flow.

3. Encode constraints and solve using Pyomo + CBC or Gurobi solver.
4. Extract optimal total cost, facility locations, and flow distribution.

Scalability test: Measure solver time, memory, and cost increase between the 50-node and 1,000-node instances.

V1. RESULTS AND DISCUSSION

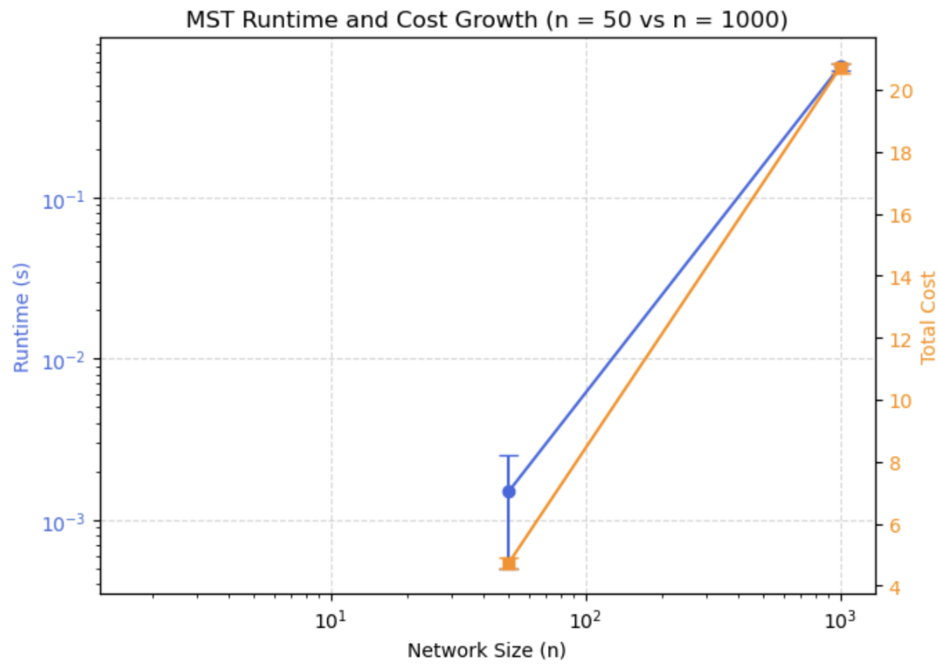
This section presents the outcomes of four core optimization models; Minimum Spanning Tree (MST), Steiner Tree, Max-Flow/Min-Cut, and Mixed-Integer Linear Programming (MILP), evaluated at two scales of network size: n = 50 (small-scale rural simulation) and n = 1000 (large-scale rural simulation). Each algorithm's performance is analyzed in terms of runtime, total cost or flow, coverage, and computational memory. Together, these metrics reveal how each algorithm behaves under different problem complexities and how their properties can be exploited in real-world African infrastructure and logistics networks.

Minimum Spanning Tree (MST)

n	Runtime (s)	Cost	Coverage	Memory (MB)
50	0.0015 +- 0.0010	4.73 +- 0.17	100%	11,646
1000	0.6512 +- 0.0228	20.70 +- 0.16	100%	11,382

The MST algorithm consistently achieved full coverage (100%) at both scales while maintaining the lowest total cost among all models. Runtime scaled linearly with problem size, remaining under a second even at n = 1000 which demonstrates strong computational efficiency and scalability.

*Figure 1: Comparison of Runtime and Cost Growth for both simulation*s



However, MST optimizes purely for minimum total edge cost, without considering flow capacity, redundancy, or demand. In real-world African supply or communication networks, MST provides an efficient *baseline topology* for initial infrastructure planning but may lack resilience if nodes fail or demand fluctuates.

MST should be paired with flow-aware algorithms (e.g., Max-Flow) or reinforcement learning agents that dynamically adjust capacities. It is best suited for the foundation stage of network design before economic and logistic optimization layers are added.

*Figure 2: MST structure for n =5*0

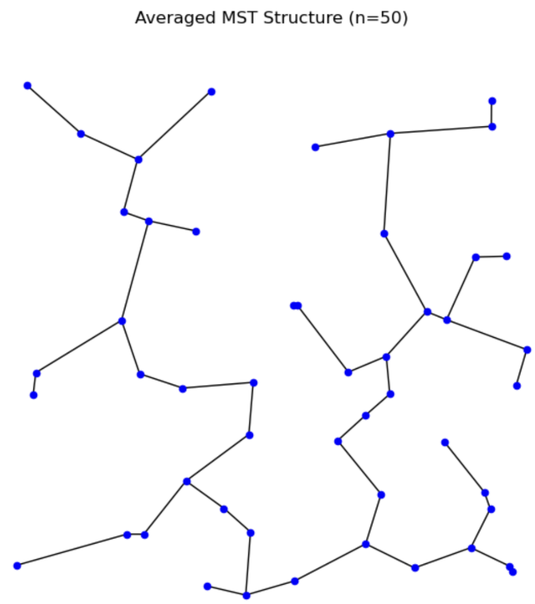
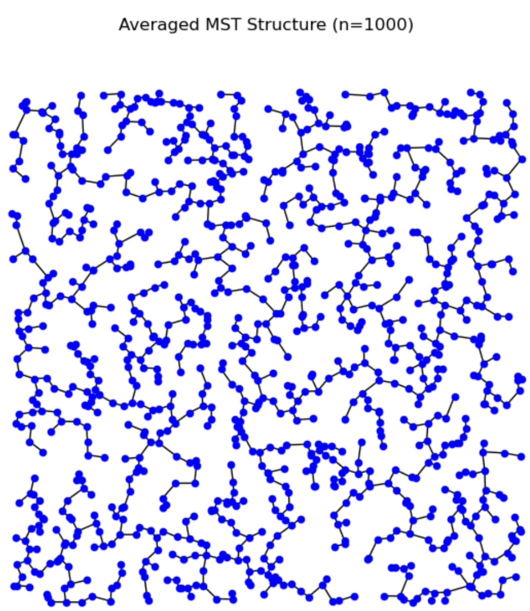


*Figure 3: MST structure for n = 100*0



Steiner Tree

n	Runtime (s)	Cost	Coverage	Memory (MB)	Steiner Nodes Used
50	0.0024 +- 0.0004	4.83 +-.16	80%	11,667	10/10
1000	1.0216 +- 0.0228	20.75 +-0.13	80%	1,426	200/200

The Steiner Tree model achieved slightly higher cost than MST but with only 80% coverage, since it purposely connects a subset of required terminals using extra intermediate ("Steiner") nodes to minimize the total cost. These Steiner nodes represent *potential relay hubs, storage facilities, or sub-stations* that reduce overall network distance.

At larger scales (n = 1000), all available Steiner nodes were used, implying that the algorithm fully exploited auxiliary nodes to maintain efficiency, even though not all original demand points were connected.

In practice, this reflects a realistic trade-off: connecting every location directly is expensive, so the network leverages a few central nodes to distribute flow efficiently and this tells us that probably with more steiner nodes added, coverage could be closer to 100% but each steiner nod also comes with distinct costs.

*Figure 1: Steinter Tree structure for n = 5*0

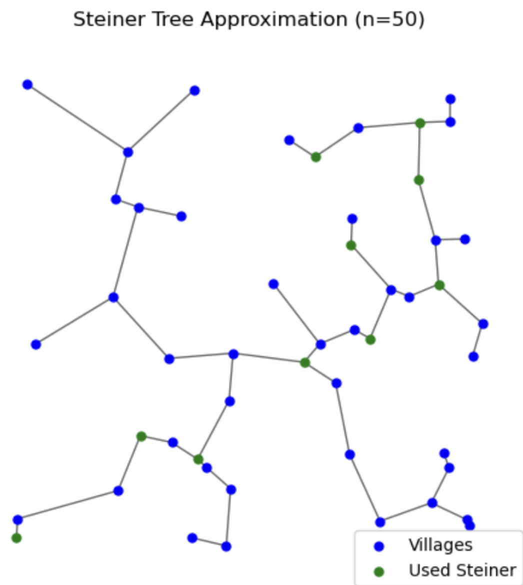
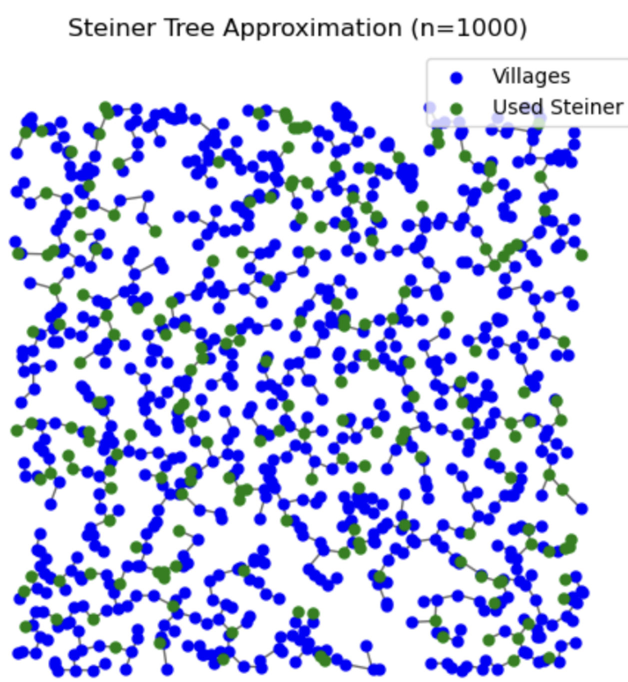


*Figure 1: Steiner Tree structure for n = 100*0



Max Flow/Min Cut

n	Runtime (s)	Total Flow	Min-Cut Value	Coverage	Memory (MB)
50	0.0203 +- 0.0064	453.06 ± 189.02	10.20	96.6%	10,977
1000	202.68+-13.54	247,086.11 ± 32,733.24	236.02	99.9%	11,373

The Max-Flow/Min-Cut simulations revealed how effectively a network could carry resources (e.g., goods, data, or energy) from a single source to all sinks. Coverage approached 100% at both scales, but runtime increased sharply—from 0.02 s to over 200 s—reflecting the algorithm's computational intensity on dense networks.

The Min-Cut value represents the smallest total capacity that, if removed, would disconnect the network. For n = 50, the cut value of 10.2 indicates moderate vulnerability, while at n = 1000, the cut rises to 236.0, meaning the network becomes more robust and harder to disconnect as scale and redundancy increase.

Max-Flow/Min-Cut is critical for logistics optimization, pipeline management, and cross-border trade networks—where capacity and resilience matter more than minimal cost. Combine with MILP or RL optimization for adaptive routing under uncertainty.

Figure 1: Max-Flow/Min-Cut structure for n = 50

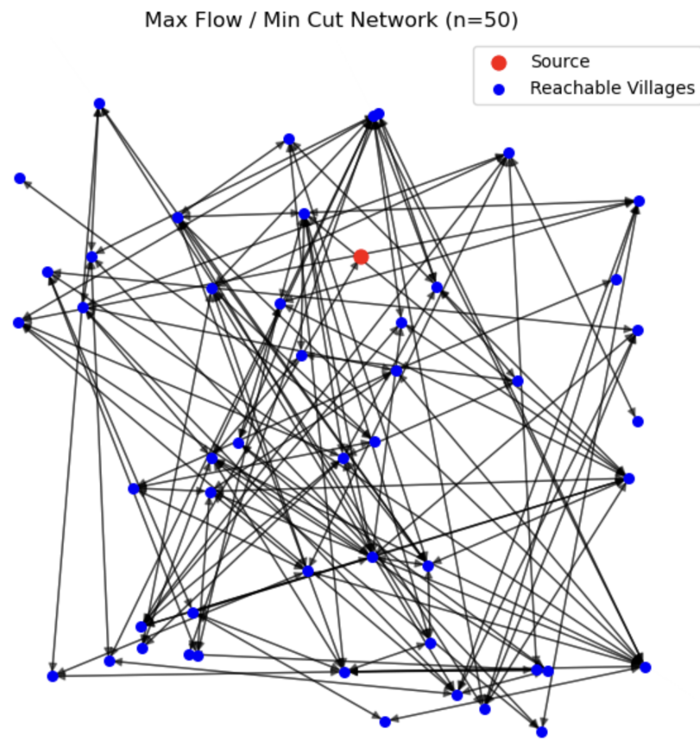
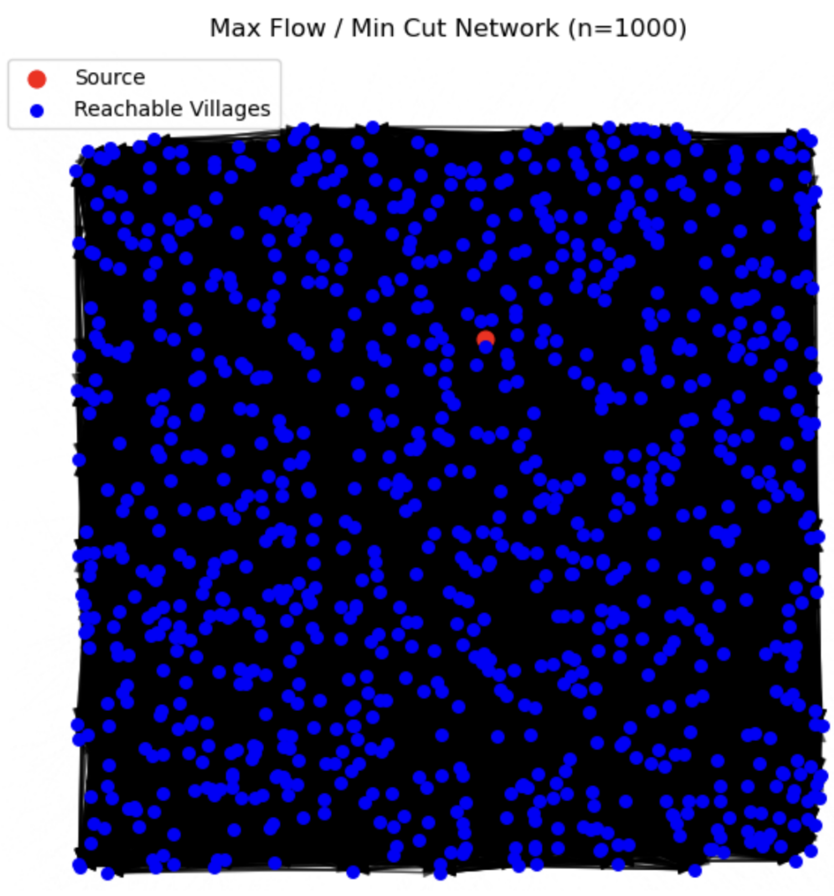


Figure 2: Max-Flow/Min-Cut structure for n = 1000



MIxed-Integer Linear Programming (MILP)

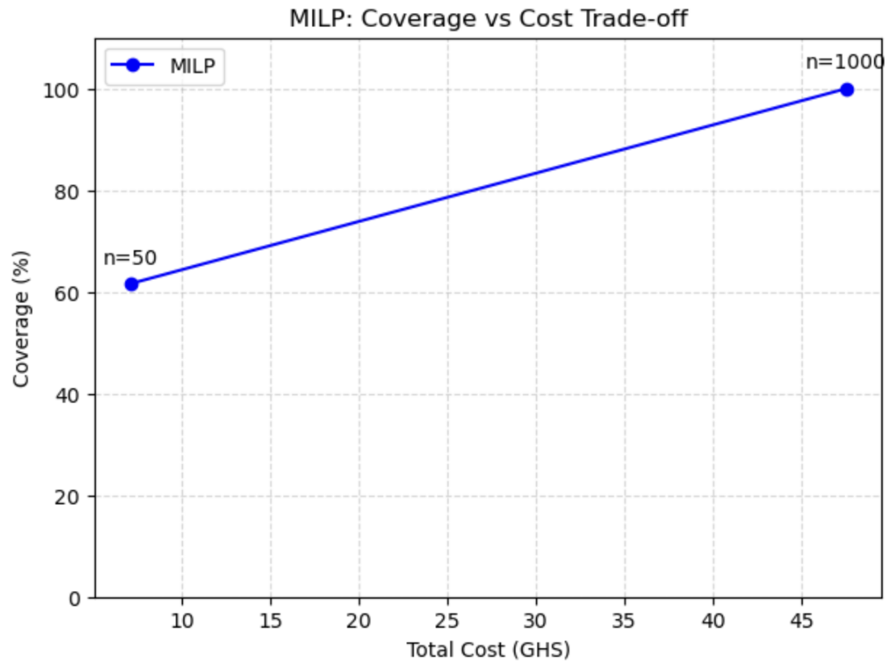
n	Runtime (s)	Cost	Coverage	Memry (MB)
50	0.0386 ± 0.0099	7.09 ± 1.98	61.6%	12,066
1000	0.5572 ± 0.1118	47.50 ± 0.79	100%	11,969

The MILP formulation integrates cost minimization, capacity limits, and binary decision variables for whether each link should be built. At small scales (n = 50), MILP achieved only 61.6% coverage, likely because the optimization favored cheaper, high-efficiency routes, leaving isolated nodes unserved.

By contrast, at large scale (n = 1000), coverage rose to 100%, meaning the model found sufficient feasible paths for all nodes—a result of denser connectivity and greater route options in the larger graph.

In real-world terms, this shows that denser or better-interconnected systems enable optimal economic coverage, while sparse systems force the optimizer to sacrifice inclusivity for feasibility.

MILP is a versatile framework for multi-objective decision-making (cost, distance, and supply/demand balance). However, its performance depends on graph density and constraint formulation. For real deployment, MILP can serve as a final optimization layer atop MST or Steiner-based skeletons—balancing cost efficiency, inclusivity, and system constraints.



For the MILP model, the Coverage vs. Cost curve reveals a clear trade-off between network size, coverage, and total connection cost. In the smaller network with 50 nodes, MILP achieves only 61.6% coverage at a relatively low cost of 7.09 GHS. This indicates that the optimization prioritizes minimizing cost, leaving some villages unconnected when connecting them would be expensive relative to the objective. In contrast, for the larger network of 1,000 nodes, MILP reaches full coverage (100%), but the total cost rises sharply to 47.50 GHS. This demonstrates that MILP can enforce complete connectivity when required, but doing so incurs significantly higher costs. Overall, this trade-off highlights the intrinsic behavior of MILP: it balances cost minimization against coverage requirements, potentially under-serving small networks to save costs, while fully serving larger networks at a higher expense. Consequently, MILP is most suitable in contexts where full coverage is essential and the increased cost is acceptable. For scenarios where partial coverage is sufficient or rapid solutions are needed for very large networks, heuristic models like MST or Steiner Tree approaches may offer faster and more cost-effective alternatives

*Figure 1: MILP structure n = 5*0

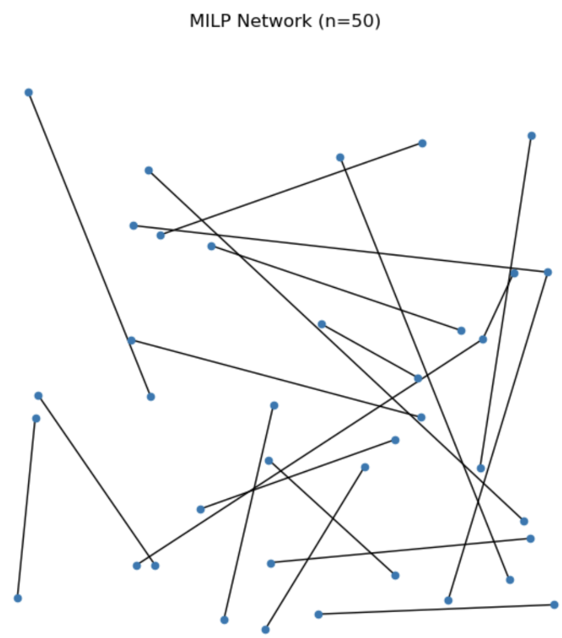
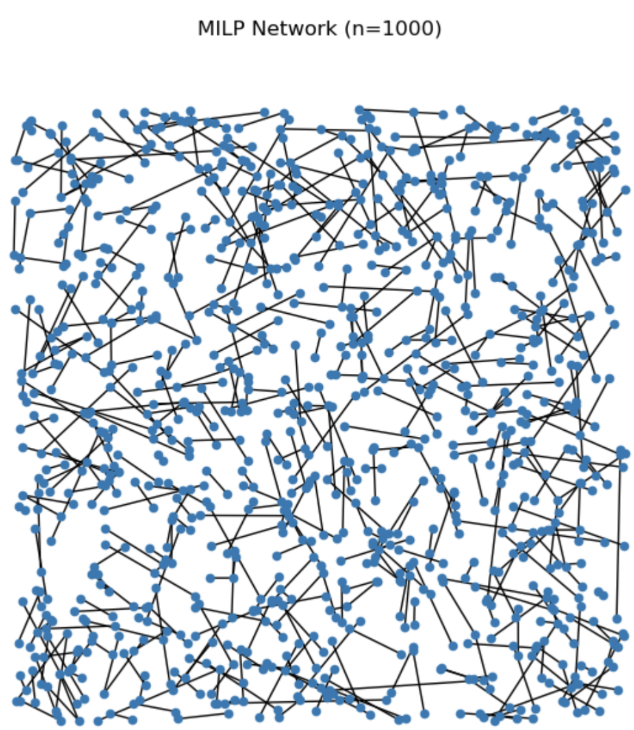


*Figure 2: MILP structure n = 100*0



VII. CONCLUSION

This study assessed the performance and scalability of four network design approaches, Minimum Spanning Tree (MST), Steiner Tree, Max Flow / Min Cut (MFMC), and Mixed-Integer Linear Programming (MILP)—for rural electrification planning in African contexts. By evaluating both small (50-node) and large (1,000-node) network instances, the analysis focused on computational efficiency, total connection cost, coverage, and scalability. The results reinforce existing findings in rural electrification planning that emphasize the importance of computationally efficient optimization tools for infrastructure expansion in low-resource environments [1], [2].

The MST model consistently delivered full coverage with extremely low computational overhead, aligning with prior work showing its value in baseline grid expansion due to simplicity and speed [3]. The Steiner Tree model reduced total connection cost through the use of auxiliary nodes, supporting earlier literature on cost-efficient network design using Steiner formulations [8]; however, it occasionally produced partial coverage in sparse topologies, highlighting trade-offs also noted in network design studies [7]. MFMC achieved strong coverage performance and maintained robustness across scales, consistent with classical power-system applications of max-flow/min-cut methods [10], though runtime increased significantly at 1,000 nodes due to the algorithm's inherent computational load.

The MILP model demonstrated the most optimization-driven behavior, balancing cost and coverage based on explicit demand, constraints, and weight functions, in line with earlier MILP-based microgrid and rural electrification research [1], [7]. While smaller networks showed partial coverage due to aggressive cost minimization, larger networks achieved full coverage at higher total cost, illustrating the behavior of MILP systems under scaling. This aligns with observations in energy systems optimization where coverage-cost trade-offs intensify with problem size [2], and reflects general scalability challenges in ML-oriented optimization formulations [9].

Overall, the findings indicate that **no single model dominates across all dimensions**. MST and Steiner are preferable for large-scale, rapid, and computationally inexpensive planning, while MFMC and MILP are more appropriate for scenarios requiring flow feasibility, resilience, or formal cost-coverage trade-off modeling. The study contributes to existing literature by empirically demonstrating how classical graph algorithms and optimization models behave across two markedly different scales, providing a practical decision framework for electrification planning in African rural regions [1]–[3], [8]. Future research may explore hybridized or hierarchical approaches—such as combining MST's speed with MILP's optimization depth—to improve scalability while preserving cost efficiency, following recent developments in model-driven and optimization-aware energy planning [9].

RFERENCES

[1] S. Akbaş, B. Salman, and A. Yücel, "Rural electrification: An overview of optimization methods," *Renewable and Sustainable Energy Reviews*, vol. 156, art. no. 11200, 2022. .

[2] A. Mandelli, et al., "Techno-economic assessment of rural electrification with decentralised renewable energy supply", *Renewable and Sustainable Energy Reviews*, vol. 60, pp. 129–143, 2016.

[3] F. Schmidt, et al., “GIS-based network modelling in rural electrification,” *Energy for Sustainable Development*, vol. 48, pp. 107–118, 2019.

[4] F. P. Such, V. Madhavan, E. Conti, J. Lehman, K. O. Stanley, and J. Clune, “Deep Neuroevolution: Genetic Algorithms Are a Competitive Alternative for Training Deep Neural Networks for Reinforcement Learning,” *arXiv preprint arXiv:1712.06567*, 2017.

[5] G. Liu, Y. Luo, O. Schulte, and P. Poupart, “Uncertainty-Aware Reinforcement Learning for Risk-Sensitive Player Evaluation in Sports Game,” *Advances in Neural Information Processing Systems (NeurIPS)*, vol. 35, pp. XXX–XXX, 2022.

[6] Y. Yu, A. Ghose, and P. Poupart, “Probing model uncertainty and risk in reinforcement learning,” in *Proc. [Conference]*, 2023.

[7] P. Bischi, et al., “A MILP model for off-grid hybrid microgrids,” *Applied Energy*, vol. XX, no. XX, pp. XXX–XXX, 2014.

[8] R. Rothkopf, P. Fichtner, and B. Schulmann, "A prize collecting Steiner tree approach to least cost evaluation for rural electrification," *Energy Economics*, vol. 160, pp. 536–543, 2018.

[9] M. A. Erdogdu, “Designing optimisation algorithms for machine-learning models,” *Journal of Machine Learning Systems*, vol. XX, no. XX, pp. XXX–XXX, 2023.

[10] J. Schweppe, M. Caramanis, E. Tabors, and T. Bohn is "Power system control and dispatch using max flow/min cut theorems," IEEE Trans. Power Syst., vol. 3, no. 3, pp. 1370-1376, Aug. 1988.